Name Key Class Date

3-4

Solving Systems Algebraically

COMMON CORE

CC.8.EE.8b
CC.8.EE.8c

Essential question: *How can you solve a system of equations algebraically?*

You have already seen how to solve a system of equations by graphing. Now you will learn to solve systems using algebra.

1 EXAMPLE Solving Systems Algebraically

Solve each system algebraically.

A $\begin{cases} y = 7x + 10 \\ y = 9x + 38 \end{cases}$

$7x + 10 = 9x + 38$

$-7x \quad -7x$

$10 = 2x + 38$

$-38 \quad -38$

$-28 = 2x$

$-14 = x$

Substitute the expression for y given in the first equation for the value of y in the second equation.

Then use properties of equality to solve the equation for x.

$y = 7(-14) + 10$

$y = -98 + 10$

$y = -88$

Substitute the value of x into one of the original equations to solve for y.

The solution of the system is $(-14, -88)$.

B $\begin{cases} 3x + 4y = 31 \\ 2x - y = 6 \end{cases}$

$2x - y = 6$

$-2x \quad -2x$

$-y = 6 - 2x$

$-y(-1) = (6 - 2x)(-1)$

$y = -6 + 2x$

Solve one equation for one of the variables. Because y is by itself in the second equation, solving that equation for y is a good place to start.

$3x + 4y = 31$

$3x + 4(-6 + 2x) = 31$

$3x + (-24) + 8x = 31$

$11x - 24 = 31$

$+24 \quad +24$

$11x = 55$

$x = 5$

Substitute the expression for y into the first equation and solve for x.

$2x - y = 6$
$2(5) - y = 6$
$10 - y = 6$
$-y = -4$
$y = 4$

Substitute the value of x into one of the original equations to solve for y.

The solution of the system is (5, 4).

REFLECT

1. How can you check your answer?

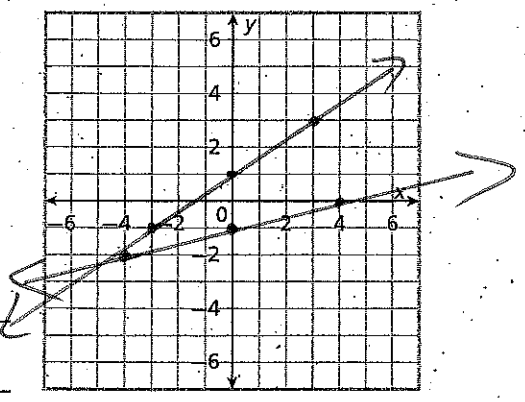
Put the x & y values back into both equations. Both statements should be TRUE.

You can use a graph to estimate the solution of a system of equations before solving algebraically.

2 EXAMPLE Using a Graph to Estimate the Solution of a System

Solve the system $\begin{cases} x - 4y = 4 \\ 2x - 3y = -3 \end{cases}$ $\rightarrow y = \frac{1}{4}x + -1$ $\rightarrow y = \frac{2}{3}x + 1$

A Sketch a graph of each linear function by substituting some values for x and generating values of y.



B The lines appear to intersect near (−5, −2). How can you tell whether (−5, −2) is the solution of the system?

check it

$x - 4x = 4$

$-5 - 4(-2) = 4$

$-5 + 8 = 4$

$3 \neq 4$ So it's not a solution

C Solve the system algebraically.

$x - 4y = 4$
$x = 4y + 4$

$2(4y + 4) + 3y = -3$
$8y + 8 + 3y = -3$
$5y = -11$
$y = -\frac{11}{5}$

$x - 4(-\frac{11}{5}) = 4$
$x + \frac{44}{5} = \frac{20}{5}$
$x = -\frac{24}{5}$

The solution is $(-\frac{24}{5}, -\frac{11}{5})$.

D Use the estimate you made using the graph to judge the reasonableness of your solution.

$-\frac{24}{5} \approx -5$ & $-\frac{11}{5} \approx -2$. This seems reasonable.

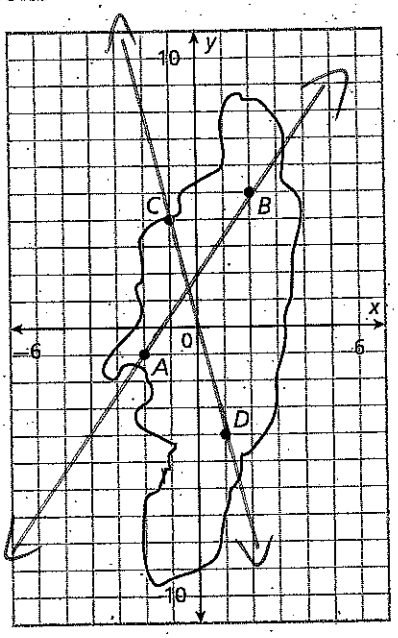
REFLECT

2. How can you determine that the system $\begin{cases} 5x - 2y = 8 \\ 5x - 2y = -3 \end{cases}$ has no solution without graphing or using algebraic methods?

$5x - 2y$ cannot equal two different #s

3 EXAMPLE Problem Solving with Systems of Equations

Aaargh! There's pirate treasure to be found,
So search on the island, all around.
Draw a line through A and B.
Then a second line through C and D.
Dance a jig, "X" marks the spot,
If the lines intersect, that's the treasure's plot!



A Give the coordinates of each point and find the slope of the line through each pair of points.

A: (-2, -1) C: (-1, 4)

B: (2, 5) D: (1, -4)

Slope: $\frac{5 - -1}{2 - -2} = \frac{6}{4} = \frac{3}{2}$

Slope: $\frac{4 - -4}{-1 - 1} = \frac{8}{-2} = -4$

B Use the slopes of the lines to determine whether they will intersect.

Different slopes which means they will intersect

C Write equations in slope-intercept form describing the line through points A and B and the line through points C and D.

Line through A and B:

$5 = \frac{3}{2} \cdot 2 + b$

$5 = 3 + b$

$b = 2$

$y = \frac{3}{2}x + 2$

Line through C and D:

$-4 = -4(1) + b$

$-4 = -4 + b$

$b = 0$

$y = -4x$

D Solve the system algebraically.

$\frac{3}{2}x + 2 = -4x$

$3x + 4 = -8x$

$11x = -4$

$x = \frac{-4}{11}$

$y = -4\left(\frac{-4}{11}\right)$

$y = \frac{16}{11}$

The solution is $\left(\frac{-4}{11}, \frac{16}{11}\right)$.

PRACTICE

Solve each system of equations algebraically.

1. $\begin{cases} y = \frac{2}{3}x - 5 \\ y = -x + 10 \end{cases}$ (9, 1)

2. $\begin{cases} 3x + 2y = 9 \\ y = 4x - 1 \end{cases}$ (1, 3)

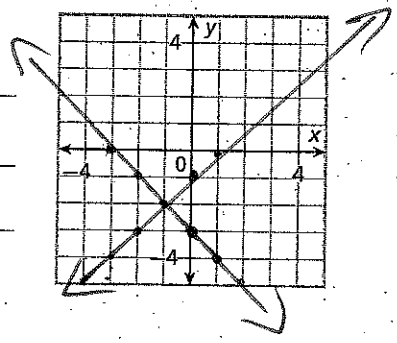
3. $\begin{cases} 5x - 2y = 4 \\ 2x - y = 1 \end{cases}$ (2, 3)

4. Error Analysis Zach solves the system $\begin{cases} x + y = -3 \\ x - y = 1 \end{cases}$ and finds the solution (1, −2).

Use a graph to explain whether Zach's solution is reasonable.

$y = -x + -3$ The x-value

$y = x + -1$ should be neg.



5. Error Analysis Angelica solves the system $\begin{cases} 3x - y = 0 \\ \frac{1}{4}x + \frac{3}{4}y = \frac{5}{2} \end{cases}$ and finds the solution (1, 3). Use substitution to explain whether Angelica's solution is correct.

$3(1) - 3 = 0$

$3 - 3 = 0$

$0 = 0$ ✓

$\frac{1}{4}(1) + \frac{3}{4}(3) = \frac{5}{2}$

$\frac{1}{4} + \frac{9}{4} = \frac{5}{2}$

$\frac{10}{4} = \frac{10}{4}$ ✓

Angelo bought apples and bananas at the fruit stand. He bought 20 pieces of fruit and spent $11.50. Apples cost $0.50 and bananas cost $0.75 each.

6. Write a system of equations to model the problem. (Hint: One equation will represent the number of pieces of fruit. A second equation will represent the money spent on the fruit.)

$x + y = 20$

$.5x + .75y = 11.5$

x = # of apples y = # of bananas

7. Solve the system algebraically. Tell how many apples and bananas Angelo bought.

(14, 6) 14 app. 6 ban.