Scatter Plots and Predictions

Essential question: *How can you use a trend line to make a prediction from a scatter plot?*

CC.8.SP2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

CC.8.SP3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Vocabulary
trend line

Prerequisites
Scatter plots and association

Math Background
When you graph bivariate data in a scatter plot and you notice that the data have a positive or negative association, you can use a trend line to make predictions about the values that are not graphed. The accuracy of predictions made by using a trend line can be judged by how closely the line fits the data. The equation of the trend line serves as a model of the data, and properties of the trend line have meanings associated with the data.

INTRODUCE

Briefly review with students scatter plots that have positive, negative, and no association. Tell students that in this lesson they will use scatter plots to write equation models for situations involving two-variable data and use the models to make predictions.

TEACH

1 EXPLORE

Questioning Strategies
- Why do you disregard outliers when drawing a trend line? **because they do not fit the trend**
- Do you think it is possible to draw a trend line for a scatter plot with no association? Explain. **No; it does not show a trend.**

Technology
Show students how to enter two-variable data into lists in a graphing calculator, display a scatter plot, and graph a trend line, or *line of best fit*. Explain that the calculator graphs the best trend line possible based on calculations involving the minimum distance possible between data points and the trend line.

2 EXAMPLE

Questioning Strategies
- What is the first step in writing an equation of a trend line? **Choose two points that the trend line will go through, and use them to find the slope.**
- How can you use the slope to write an equation in the form $y = mx + b$ for the trend line? **Substitute the slope value for m and the coordinates of one of the points on the line for x and y into $y = mx + b$. Solve for b. Write the equation in the form $y = mx + b$, where x is the slope and b is the y-intercept.**
- If the chapters were on the vertical axis and the pages were on the horizontal axis, what would be the value of the slope and what would it represent? **The slope of the line would be $\frac{1}{10}$, representing a rate of $\frac{1}{10}$, or 0.1, of a chapter per page.**

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Name_____ Class_____ Date_____   6-2

Scatter Plots and Predictions

COMMON CORE CC.8.SP2 CC.8.SP3

Essential question: *How can you use a trend line to make a prediction from a scatter plot?*

When a scatter plot shows a linear association, you can use a line to model the relationship between the variables. A trend line is a straight line that comes closest to the points on a scatter plot.
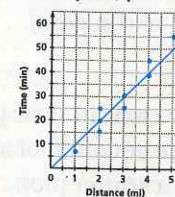
1 EXPLORE   Drawing a Trend Line

Joyce is training for a 10K race. For some of her training runs, she records the distance she ran and how many minutes she ran.

A   Make a scatter plot of Joyce's running data.

Distance (mi)	Time (min)
4	38
2	25
1	7
2	16
3	26
5	55
2	20
4	45
3	31

B   To draw a trend line, use a straight edge to draw a line that has about the same number of points above and below it. Ignore any outliers.

C   Use your trend line to predict how long it would take Joyce to run 4.5 miles. **about 45 minutes**

REFLECT

1a. How well does your trend line fit the data? **All the data points are close to the line. The data shows a strong linear association, so the line should fit very well.**

1b. Do you think you can use a scatter plot that shows no association to make a prediction? Explain your answer. **No; no association means that there is no relationship between the variables and the scatter plot shows no pattern.**

2 EXAMPLE   Finding the Equation of a Trend Line

The scatter plot shows the relationship between the number of chapters and the total number of pages for several books. Draw a trend line, write an equation for the trend line, and describe the meanings of the slope and y-intercept.
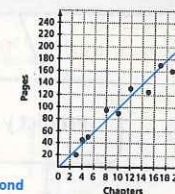
A   Draw a trend line. It will be easier to write an equation for the line if it goes through two of the data points. (Hint: Use (5, 50) as one of the points.) Identify another point that the trend line goes through: $\left(17, 170 \right)$ **Answers may vary based on the second point selected.**

B   What type(s) of association does the scatter plot show? **positive; linear**

C   Do you expect the slope of the line to be positive or negative? **positive**

D   Find the slope of the trend line.
$$m = \frac{170 - 50}{17 - 5} = \frac{120}{12} = 10$$

E   Use the equation $y = mx + b$, the slope, and the point (5, 50). Substitute values for y, m, and x into the equation and solve for b.

$y = mx + b$	
$50 = 10 \cdot 5 + b$	Substitute for y, m, and b.
$50 = 50 + b$	Simplify on the right side.
$-50 \quad -50$	Subtract the number that is added to b from both sides.
$0 = b$	

Use your slope and y-intercept values to write an equation in slope-intercept form.
$y = 10 x + 0$

F   What is the meaning of the slope in this situation? **There is an average of 10 pages per chapter; an additional chapter is associated with 10 additional pages.**

G   What is the meaning of the y-intercept in this situation? **the number of pages in a book with 0 chapters (0)**

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Questioning Strategies

- How can you use the equation of a trend line to make predictions? **You can substitute a value for either x or y, and solve for the value of the other variable. Then interpret the meaning for the context.**

- What is the difference between interpolation and extrapolation? **Interpolation involves the prediction of values by using data points from within the boundaries of the data used to write the trend line. Extrapolation involves the prediction of values by using data points that lie beyond those boundaries.**

MATHEMATICAL PRACTICE   **Highlighting the Standards**

This Explore is an opportunity to address Standard 1 (Make sense of problems and persevere in solving them). Students use scatter plots and trend lines to interpret situations involving two unknowns, to examine for possible associations between the variables, and to help them make predictions for the data in the scatter plot when there is a reasonably strong association. This multi-step process allows students to practice perseverance in problem solving.
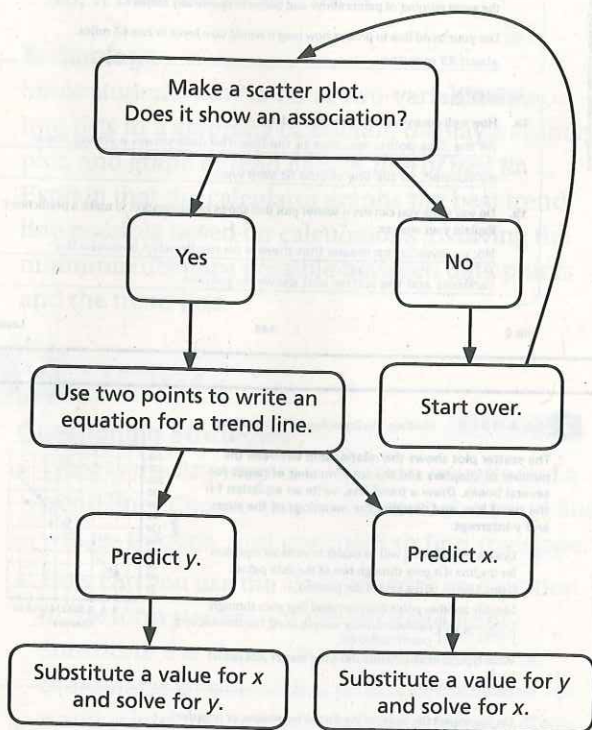

CLOSE

Essential Question

How can you use a trend line to make a prediction from a scatter plot?

When a scatter plot shows a reasonably strong positive or negative association between two variables, draw a trend line as closely fitting as possible through the points. Then write an equation for that line, and use it to make predictions by substituting and solving.

Summarize

Have students make a flow chart in their journals to show the steps for using the equation of a trend line to make a prediction from a scatter plot.

Make a scatter plot. Does it show an association?

→ Yes → Use two points to write an equation for a trend line.

→ No → Start over.

Predict y. → Substitute a value for x and solve for y.

Predict x. → Substitute a value for y and solve for x.

PRACTICE

Where skills are taught	Where skills are practiced
1 EXPLORE	EXS. 1–2, 7
2 EXAMPLE	EX. 3
3 EXPLORE	EXS. 4–6

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When you use a trend line or its equation to predict a value between data points that you already know, you *interpolate* the predicted value. When you make a prediction that is outside the data that you know, you *extrapolate* the predicted value.

3 EXPLORE   Making Predictions   **Answers may vary depending on the equation found in the previous Example.**

Refer to the scatter plot and trend line in **2**.

A Use the equation of the trend line to predict how many pages would be in a book with 26 chapters.

Is this prediction an example of interpolation or extrapolation?

extrapolation

$y =$ **10x**   *Write the equation for your trend line.*

$y =$ **10(26)**   *Substitute the number of chapters for x.*

$y =$ **260**   *Simplify.*

I predict that a book with 26 chapters would have **260** pages.

B Use the equation of the trend line to predict how many pages would be in a book with 14 chapters.

Is this prediction an example of interpolation or extrapolation?

interpolation

$y =$ **10x**   *Write the equation for your trend line.*

$y =$ **10(14)**   *Substitute the number of chapters for x.*

$y =$ **140**   *Simplify.*

I predict that a book with 14 chapters would have **140** pages.

REFLECT

3a. How well do your new points fit the original data?

**They fall close to the original data, so they fit well.**

3b. Do you think that extrapolation or interpolation is more accurate? Explain.

Possible answer: Interpolation is more accurate because the predicted value fits between known points where the trend is known. There is no guarantee that a trend will continue beyond the known data points.

PRACTICE

Angela recorded the price of different number of ounces of bulk grains. She made a scatter plot of her data. Use the scatter plot for 1–5.

1. Draw a trend line for the scatter plot.
Answers for 1–5 may vary slightly.

2. How do you know whether your trend line is a good fit for the data?

Most of the data points are close to the trend line and there is about the same number of points above and below the line.
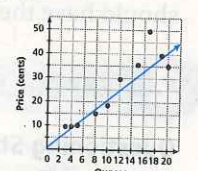
3. Write an equation for your trend line. **y = 2x + 1**

4. Use the equation for your trend line to interpolate the price of 7 ounces.

15¢

5. Use the equation for your trend line to extrapolate the price of 50 ounces.

101¢; $1.01

6. A scatter plot shows the relationship between a baby's length and age. Why might an extrapolated data point not be very accurate?

Sample answer: Babies grow very quickly at first, but as they age, their growth rate decreases. Eventually, a person's length (height) stops increasing.

7. Error Analysis Carl graphed the data shown in the scatter plot and then drew a trend line. Why is a trend line not a good fit for this data?

The data does not show a linear association, so a linear model is not a good fit.

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Notes