Name: Key

Substitution

What does the word "substitution" mean?

So far, we have been solving systems of linear equations by graphing each equation and finding the point of intersection. There is another method called solving systems by substitution. Just like you can substitute onion rings for fries at a restaurant, or a substitute teacher can teach your class, solving by substitution involves one 'thing' taking the place of another equivalent 'thing'. In this case, our 'things' are variables.

Consider some examples...

1. $y = x - 5$
   $2x + y = 1$

$2x + (x-5) = 1$
$3x - 5 = 1$
$3x = 6$
$x = 2$

$y = 2 - 5$
$y = -3$

#1 Solution: $(2, -3)$

2. $2x + y = 1$
   $y = 2x - 3$

$2x + 2x - 3 = 1$
$4x = 4$
$x = 1$

$y = 2(1) - 3$
$y = 2 - 3$
$y = -1$

#2 Solution: $(1, -1)$

Now you try some...

3. $12x - 5y = 30$
   $y = 2x - 6$

$12x - 5(2x-6) = 30$
$12x + {}^-10x + 30 = 30$
$2x = 0$
$x = 0$

$y = 2(0) - 6$
$y = 0 - 6$
$y = -6$

#3 Solution: $(0, -6)$

4. $x + y = 14$
   $y = 6x$

$x + 6x = 14$
$7x = 14$
$x = 2$

$y = 6(2)$
$y = 12$

#4 Solution: $(2, 12)$

5. $y = \frac{1}{2}x - 3$
$\frac{1}{2}x + y = 1$

$\frac{1}{2}x + \frac{1}{2}x - 3 = 1$

$x = 4$

$y = \frac{1}{2}(4) - 3$

$y = 2 - 3$

$y = -1$

#5 Solution: $(4, -1)$

6. x = 3y
2x + 4y = 20

$2(3y) + 4y = 20$

$6y + 4y = 20$

$10y = 20$

$y = 2$

$x = 3(2)$

$x = 6$

#6 Solution: $(6, 2)$

Now we need to solve one of the equations for x or y first.

7. 3x + y = -1
x - y = -3 $\rightarrow x = y + {}^-3$

$3(y + {}^-3) + y = -1$

$3y + {}^-9 + y = -1$

$4y = 8$

$y = 2$

$3x + 2 = -1$

$3x = -3$

$x = -1$

#7 Solution: $(-1, 2)$

8. 3y + 2x = 4
-6x - y = -4 $\rightarrow y = -6x + 4$

$3(-6x + 4) + 2x = 4$

$-18x + 12 + 2x = 4$

$-16x = -8$

$x = \frac{1}{2}$

$3y + 2(\frac{1}{2}) = 4$

$3y + 1 = 4$

$3y = 3$

$y = 1$

#8 Solution: $(\frac{1}{2}, 1)$

9. 3x + 2y = 16
-6x + y = -7 $\rightarrow y = 6x + {}^-7$

$3x + 2(6x + {}^-7) = 16$

$3x + 12x + {}^-14 = 16$

$15x = 30$

$x = 2$

$3(2) + 2y = 16$

$6 + 2y = 16$

$2y = 10$

$y = 5$

#9 Solution: $(2, 5)$

10. 4x + y = -2 $\rightarrow y = {}^-4x + {}^-2$
-2x – 3y = 1

$-2x + {}^-3(-4x + {}^-2) = 1$

$-2x + 12x + 6 = 1$

$10x = -5$

$x = -\frac{1}{2}$

$4(-\frac{1}{2}) + y = -2$

$-2 + y = -2$

$y = 0$

#10 Solution: $(-\frac{1}{2}, 0)$

QUESTION: Do you think that you always have to solve for y? Could you solve for x instead? Why or why not?

No; yes. As long as you isolate one of the variables.

Application Problems

1. Your school is planning to bring 193 people to a competition at another school. There are eight drivers available and two types of vehicles, school buses and minivans. The school buses seat 51 people each, and the minivans seat 8 people each. How many buses and minivans will be needed?

Equations: b = # of buses

m = # of minivans

$b + m = 8$

$51b + 8m = 193$

Work: $b = -m + 8$

$51(-m+8) + 8m = 193$

$-51m + 408 + 8m = 193$

$-43m = -215$

$m = 5$

$b = 3$ (buses)
$m = 5$ minivans
$(3, 5)$

2. A rectangle is 4 times longer than it is wide. The perimeter of the rectangle is 30 cm. Find the dimensions of the rectangle.

Equations:

l = length (cm)

w = width (cm)

$2l + 2w = 30$

$l = 4w$

Work: $2w + 2(4w) = 30$

$2w + 8w = 30$

$10w = 30$

$w = 3$ cm

$l = 4(3) = 12$ cm

12cm by 3cm