

Analyzing Solutions

Essential question: How can you give examples of equations with a given number of solutions?

So far, when you solved a linear equation in one variable, you found one value of x that makes the equation a true statement. When you simplify some equations, you may find that they do not have one solution.

COMMON
CORE

CC.8.EE.7a.

1 EXPLORE Determining the Number of Solutions

Use the properties of equality to simplify each equation. Tell whether the final equation is a true statement.

A $4x - 3 = 2x + 13$

$$4x - 3 = 2x + 13$$

$$\begin{array}{r} +3 \\ \hline 4x = 2x + 16 \end{array}$$

$$\begin{array}{r} -2x \\ \hline 2x = 16 \end{array}$$

$$\begin{array}{r} -2x \\ \hline 2x = 16 \end{array}$$

$$2x = 16$$

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

$$x = 8$$

The statement is true / false.

B $4x - 5 = 2(2x - 1) - 3$

$$4x - 5 = 2(2x - 1) - 3$$

$$4x - 5 = 4x - 2 - 3$$

$$4x - 5 = 4x - 5$$

$$\begin{array}{r} -4x \\ \hline -5 = -5 \end{array}$$

$$-5 = -5$$

The statement is true / false.

C $4x + 2 = 4x - 5$

$$4x + 2 = 4x - 5$$

$$\begin{array}{r} -2 \\ \hline 4x = 4x - 7 \end{array}$$

$$4x = 4x - 7$$

$$\begin{array}{r} -4x \\ \hline 0 = -7 \end{array}$$

$$0 = -7$$

$$0 = -7$$

$$0 = -7$$

The statement is true / false.

REFLECT

- 1a. What happens when you substitute any value for x in the original equation in B? In the original equation in C?

Part B \rightarrow any # will work

Part C \rightarrow any # will make it false.

When you simplify an equation using the properties of equality, you will find one of three results.

Result	What does this mean?	How many solutions?
$x = a$	When the value of x is a , the equation is a true statement.	1
$a = a$	Any value of x makes the equation a true statement.	Infinitely many
$a = b$	There is no value of x that makes the equation a true statement.	0

2 EXPLORE Writing Equations with a Given Number of Solutions

Write a linear equation in one variable that has no solutions.

You can use the strategy of working backward:

- A Start with a false statement such as $3 = 5$. Add the same variable term to both sides.

$$3 + n = 5 + n$$

- B Next, add the same constant to both sides and combine like terms on each side of the equation.

$$\begin{array}{r} 3 + n = 5 + n \\ 7 \qquad 7 \end{array}$$

$$10 + n = 12 + n$$

- C Verify that your equation has no solutions by using properties of equality to simplify your equation.

$$\begin{array}{r} 10 + n = 12 + n \\ -n \qquad -n \end{array}$$

REFLECT

$$10 \neq 12$$

- 2a. Explain why the result of the process above is an equation with no solutions.

We started with a false statement.
 Added, subtracting, multiplying, or dividing anything to both sides will ALWAYS make a false statement for this situation.

TRY THIS!

Tell whether each equation has one, zero, or infinitely many solutions.

1. $6 + 3x = x - 8$ 1 sol. $2x = -14$
 $x = -7$
2. $8x + 4 = 4(2x + 1)$ Int. many.
 $8x + 4 = 8x + 4$

Complete each equation so that it has the indicated number of solutions.

3. No solutions: $3x + 1 = 3x + 2$
4. Infinitely many: $2x - 4 = 2x - 4$