Lesson 1: Complementary and Supplementary Angles

Student Outcomes

- Students solve for unknown angles in word problems and in diagrams involving complementary and supplementary angles.

Lesson Notes

Students review key terminology regarding angles before attempting several examples. In Lessons 1–4, students practice identifying the angle relationship(s) in each problem and then model the relationship with an equation that yields the unknown value. By the end of the four unknown angle lessons, students should be fluent in this topic and should be able to solve for unknown angles in problems that incorporate multiple angle relationships with mastery.

Solving angle problems has been a part of instruction for many years. Traditionally, the instructions are simply to solve for an unknown angle. Consider using these problems to highlight MP.4 and MP.2 to build conceptual understanding. Students model the geometric situation with equations and can readily explain the connection between the symbols they use and the situation being modeled. Ask students consistently to explain the geometric relationships involved and how they are represented by the equations they write. A final component of deep conceptual understanding is for students to assess the reasonableness of their solutions, both algebraically and geometrically.

Classwork

Opening Exercise (6 minutes)

In the tables below, students review key definitions and angle facts (statements). Students first saw this information in Grade 4, so these definitions and abbreviations should be familiar. Though abbreviations are presented, teachers may choose to require students to state complete definitions. Consider reading each statement with the *blank* and challenging students to name the missing word.

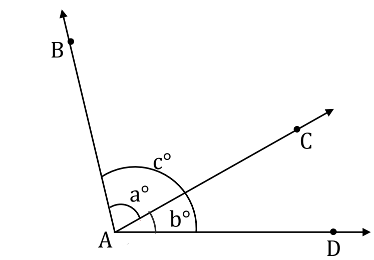
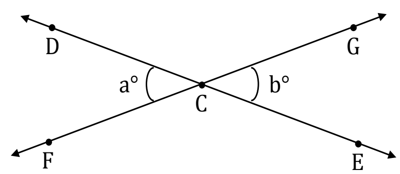
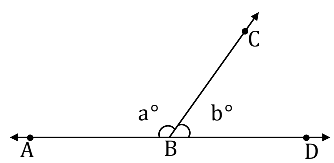
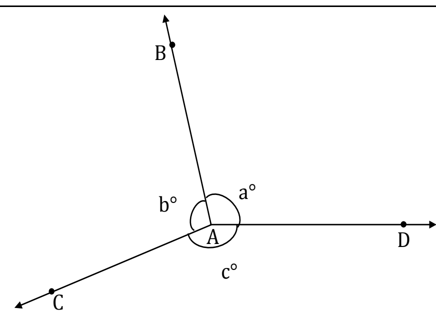
Opening Exercise

As we begin our study of unknown angles, let us review key definitions.

Term	Definition
Adjacent	**Two angles, $\angle AOC$ and $\angle COB$, with a common side $\overrightarrow{OC}$, are ____________ *angles* if C is in the interior of $\angle AOB$.**
Vertical; vertically opposite	**When two lines intersect, any two non-adjacent angles formed by those lines are called ____________ *angles*, or ____________ ____________ *angles*.**
Perpendicular	**Two lines are ____________ if they intersect in one point, and any of the angles formed by the intersection of the lines is $90°$. Two segments or rays are ____________ if the lines containing them are ____________ lines.**

Teachers should note that the *Angles on a Line* fact is stated with a diagram of two angles on a line, but the definition can be expanded to include two or more angles on a line. When two angles are on a line, they are also referred to as a *linear pair.*

Complete the missing information in the table below. In the *Statement* column, use the illustration to write an equation that demonstrates the angle relationship; use all forms of angle notation in your equations.

Angle Relationship	Abbreviation	Statement	Illustration
Adjacent Angles	*∠s add*	**The measurements of adjacent angles add.** $a + b = c$	
Vertical Angles	*vert. ∠s*	**Vertical angles have equal measures.** $a = b$	
Angles on a Line	*∠s on a line*	**If the vertex of a ray lies on a line but the ray is not contained in that line, then the sum of measurements of the two angles formed is $180°$.** $a + b = 180$	
Angles at a Point	*∠s at a point*	**Suppose three or more rays with the same vertex separate the plane into angles with disjointed interiors. Then, the sum of all the measurements of the angles is $360°$.** $a + b + c = 360$	

Note that the distinction between *an angle* and *the measurement of an angle* is not made in this table or elsewhere in the module. Students study congruence in Grade 8, where stating two angles as equal will mean that a series of rigid motions exists such that one angle will map exactly onto the other. For now, the notion of two angles being equal is a function of each angle having the same degree measurement as the other. Furthermore, while students were exposed to the m notation preceding the measurement of an angle in Module 3, continue to expose them to different styles of notation so they learn to discern meaning based on context of the situation.

Discussion (7 minutes)

Lead students through a discussion regarding the *Angles on a Line* fact that concludes with definitions of supplementary and complementary angles.

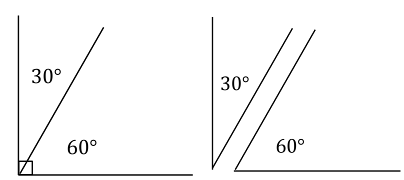
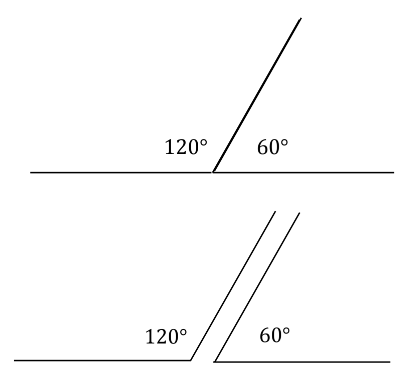
- *Angles on a Line* refers to two angles on a line whose measurements sum to $180°$. Could we use this fact if the two angles are not adjacent, but their measurements still sum to $180°$? Could we still call them angles on a line?
  - *No, it would not be appropriate because the angles no longer sit on a line.*
- It would be nice to have a term for non-adjacent angles that sum to $180°$ since $180°$ is such a special value in geometry. We can make a similar argument for two angles whose measurements sum to $90°$. It would be nice to have a term that describes two angles that may or may not be adjacent to each other whose measurements sum to $90°$.

Before students examine the following table, have them do the following exercise in pairs. Ask students to generate a pair of values that sum to $180°$ and a pair of values that sum to $90°$. Ask one student to draw angles with these measurements that are adjacent and the other student to draw angles with these measurements that are not adjacent.

Direct students to the examples of supplementary and complementary angle pairs, and ask them to try to develop their own definitions of the terms. Discuss a few of the student-generated definitions as a class, and record formal definitions in the table.

> *Scaffolding:*
>
> It may be helpful to create a prominent, permanent visual display to remind students of these definitions.

The diagrams in the lessons depict line segments only but should be interpreted as rays and lines.

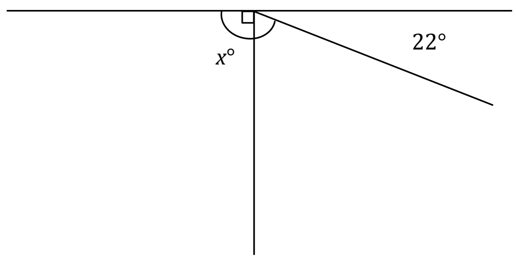
Angle Relationship	Definition	Diagram
Complementary Angles	***If the sum of the measurements of two angles is $90°$, then the angles are called complementary; each angle is called a complement of the other.***	
Supplementary Angles	***If the sum of the measurements of two angles is $180°$, then the angles are called supplementary; each angle is called a supplement of the other.***	

Exercise 1 (4 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationship in the diagram.

Exercise 1

1. **In a complete sentence, describe the relevant angle relationships in the diagram. Write an equation for the angle relationship shown in the figure, and solve for x. Confirm your answers by measuring the angle with a protractor.**



The angles $x°$ and $22°$ are supplementary and sum to $180°$.

$$x + 22 = 180$$
$$x + 22 - 22 = 180 - 22$$
$$x = 158$$

The measure of the angle is $158°$.

Example 1 (5 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationship described in the word problem.

Optional Discussion: The following four questions are a summative exercise that teachers may or may not need to do with students before Example 1 and Exercises 2–4. Review terms that describe arithmetic operations.

- *More than, increased by, exceeds by, and greater than*
- *Less than, decreased by, and fewer than*
- *Times, twice, doubled, and product of*
- *Half of* (or any fractional term, such as *one-third of*), *out of, and ratio of*

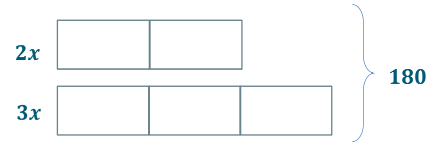
- What does it mean for two angle measurements to be in a ratio of $1:4$? Explain by using a tape diagram.
  - *Two angle measurements in a ratio of* $1:4$ *can be represented as follows.*

Once we have expressions to represent the angle measurements, they can be used to create an equation depending on the problem. Encourage students to draw a tape diagram for Example 1.

Example 1

The measures of two supplementary angles are in the ratio of $2:3$. Find the measurements of the two angles.

$$2x + 3x = 180$$
$$5x = 180$$
$$\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)180$$
$$x = 36$$



Angle 1 $= 2(36)^\circ = 72^\circ$

Angle 2 $= 3(36)^\circ = 108^\circ$

Exercises 2–4 (12 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationship described in the word problem.

Exercises 2–4

2. **In a pair of complementary angles, the measurement of the larger angle is three times that of the smaller angle. Find the measurements of the two angles.**

$$x + 3x = 90$$
$$4x = 90$$
$$\left(\frac{1}{4}\right)4x = \left(\frac{1}{4}\right)90$$
$$x = 22.5$$

Angle 1 $= 22.5^\circ$

Angle 2 $= 3(22.5)^\circ = 67.5^\circ$

3. **The measure of a supplement of an angle is 6° more than twice the measure of the angle. Find the measurement of the two angles.**

$$x + (2x + 6) = 180$$
$$3x + 6 = 180$$
$$3x + 6 - 6 = 180 - 6$$
$$3x = 174$$
$$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)174$$
$$x = 58$$

Angle 1 $= 58^\circ$

Angle 2 $= 2(58)^\circ + 6^\circ = 122^\circ$

4. The measure of a complement of an angle is $32°$ more than three times the angle. Find the measurement of the two angles.

$$x + (3x + 32) = 90$$
$$4x + 32 - 32 = 90 - 32$$
$$4x = 58$$
$$\left(\frac{1}{4}\right)4x = \left(\frac{1}{4}\right)58$$
$$x = 14.5$$

Angle 1 $= 14.5°$

Angle 2 $= 3(14.5)° + 32° = 75.5°$

Scaffolding solutions:

a. *The* $x°$ *angle and the* $22°$ *angle are supplementary and sum to* $180°$.

b. $x + 22 = 180$

c. *The equation shows that an unknown value,* x *(which is the unknown angle in the diagram) plus* 22 *is equal to* 180. *The* 22 *represents the angle that we know, which is* 22 *degrees.*

d.
$$x + 22 = 180$$
$$x + 22 - 22 = 180 - 22$$
$$x = 158$$

$x = 158$ *means that in the diagram, the missing angle measures* $158°$.

e. *The answer of* $158°$ *is correct. If we substitute* 158 *for* x, *we get* $158 + 22 = 180$, *which is a true number sentence. This is reasonable because when we look at the diagram, we would expect the angle to be obtuse.*

Scaffolding:

Students struggling to organize their solutions may benefit from the following:

- In a complete sentence, describe the relevant angle relationships in the diagram. Write an equation that represents this relationship (MP.4). Explain how the parts of the equation describe the situation (MP.2). Solve the equation, and interpret the solution. Check your answer. Is it reasonable?

Example 2 (5 minutes)

Example 2

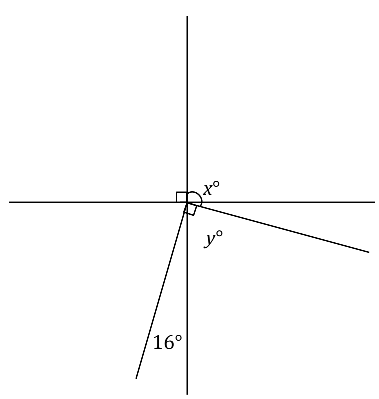
Two lines meet at a point that is also the vertex of an angle. Set up and solve an appropriate equation for x and y.

$$16 + y = 90$$
$$16 - 16 + y = 90 - 16$$
$$y = 74$$

Complementary angles

$$x + (74) = 180$$
$$x + 74 - 74 = 180 - 74$$
$$x = 106$$

Supplementary angles



Closing (1 minute)

- To determine the measurement of an unknown angle, we must identify the angle relationship(s) and then model the relationship with an equation that yields the unknown value.
- If the sum of the measurements of two angles is $90°$, the angles are complementary angles, and one is the complement of the other.
- If the sum of the measurements of two angles is $180°$, the angles are supplementary angles, and one is the supplement of the other.

Lesson Summary

- **Supplementary angles are two angles whose measurements sum to $180°$.**
- **Complementary angles are two angles whose measurements sum to $90°$.**
- **Once an angle relationship is identified, the relationship can be modeled with an equation that will find an unknown value. The unknown value may be used to find the measure of the unknown angle.**

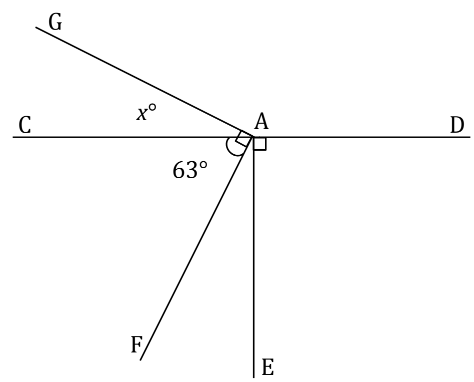
Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 1: Complementary and Supplementary Angles

Exit Ticket

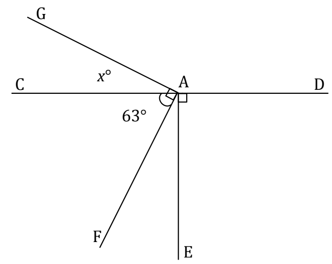
1. Set up and solve an equation for the value of x. Use the value of x and a relevant angle relationship in the diagram to determine the measurement of $\angle EAF$.



2. The measurement of the supplement of an angle is 39° more than half the angle. Find the measurement of the angle and its supplement.

Exit Ticket Sample Solutions

1. **Set up and solve an equation for the value of x. Use the value of x and a relevant angle relationship in the diagram to determine the measurement of $\angle EAF$.**



$$x + 63 = 90$$
$$x + 63 - 63 = 90 - 63$$
$$x = 27$$

$\angle CAG$ and $\angle EAF$ are the complements of $63°$. The measurement of $\angle CAG$ is $27°$; therefore, the measurement of $\angle EAF$ is also $27°$.

2. **The measurement of the supplement of an angle is $39°$ more than half the angle. Find the measurement of the angle and its supplement.**

$$x + \left(\frac{1}{2}x + 39\right) = 180$$
$$1.5x + 39 = 180$$
$$1.5x + 39 - 39 = 180 - 39$$
$$1.5x = 141$$
$$1.5x \div 1.5 = 141 \div 1.5$$
$$x = 94$$

The measurement of the angle is $94°$.

The measurement of the supplement is $\frac{1}{2}(94)° + 39° = 86°$.

OR

$$x + \left(\frac{1}{2}x + 39\right) = 180$$
$$\frac{3}{2}x + 39 = 180$$
$$\frac{3}{2}x + 39 - 39 = 180 - 39$$
$$\frac{3}{2}x = 141$$
$$\left(\frac{2}{3}\right)\left(\frac{3}{2}x\right) = \left(\frac{2}{3}\right)141$$
$$x = 94$$

The measurement of the angle is $94°$.

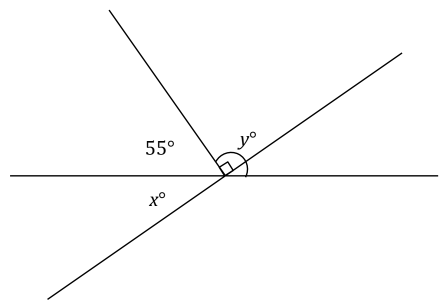
The measurement of the supplement is $\frac{1}{2}(94)° + 39° = 86°$.

Problem Set Sample Solutions

1. Two lines meet at a point that is also the endpoint of a ray. Set up and solve the appropriate equations to determine x and y.

$$x + 55 = 90 \qquad \textit{Complementary angles}$$
$$x + 55 - 55 = 90 - 55$$
$$x = 35$$

$$55 + y = 180 \qquad \textit{Supplementary angles}$$
$$55 - 55 + y = 180 - 55$$
$$y = 125$$



Scaffolding:

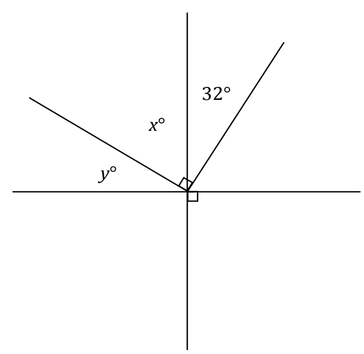
As shown in Exercise 4, some students may benefit from a scaffolded task. Use the five-part scaffold to help organize the question for those students who might benefit from it.

2. Two lines meet at a point that is also the vertex of an angle. Set up and solve the appropriate equations to determine x and y.

$$y + x = 90 \qquad \textit{Complementary angles}$$

$$x + 32 = 90 \qquad \textit{Complementary angles}$$
$$x + 32 - 32 = 90 - 32$$
$$x = 58$$

$$y + (58) = 90 \qquad \textit{Complementary angles}$$
$$y + 58 - 58 = 90 - 58$$
$$y = 32$$

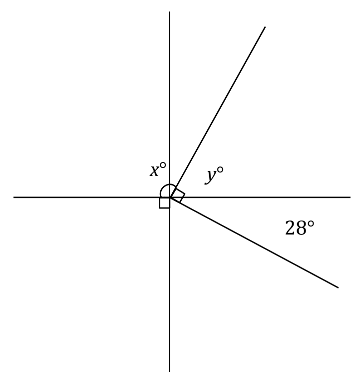


3. Two lines meet at a point that is also the vertex of an angle. Set up and solve an appropriate equation for x and y.

$$x + y = 180 \qquad \textit{Supplementary angles}$$

$$28 + y = 90 \qquad \textit{Complementary angles}$$
$$28 - 28 + y = 90 - 28$$
$$y = 62$$

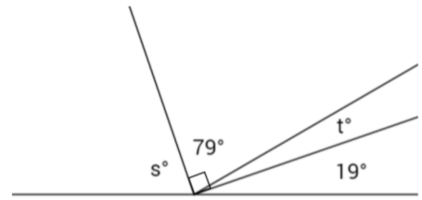
$$x + (62) = 180 \qquad \textit{Supplementary angles}$$
$$x + 62 - 62 = 180 - 62$$
$$x = 118$$



4. **Set up and solve the appropriate equations for s and t.**

$$79 + t = 90 \quad \textit{Complementary angles}$$
$$79 - 79 + t = 90 - 79$$
$$t = 11$$

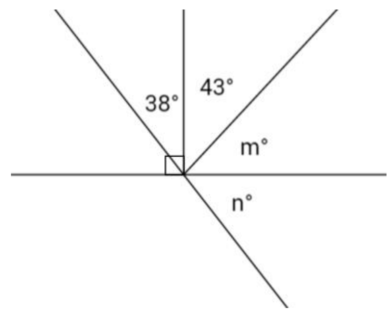
$$19 + (11) + 79 + s = 180 \quad \textit{Angles on a line}$$
$$109 + s = 180$$
$$109 - 109 + s = 180 - 109$$
$$s = 71$$



5. **Two lines meet at a point that is also the endpoint of two rays. Set up and solve the appropriate equations for m and n.**

$$43 + m = 90 \quad \textit{Complementary angles}$$
$$43 - 43 + m = 90 - 43$$
$$m = 47$$

$$38 + 43 + (47) + n = 180 \quad \textit{Angles on a line}$$
$$128 + n = 180$$
$$128 - 128 + n = 180 - 128$$
$$n = 52$$



6. **The supplement of the measurement of an angle is $16°$ less than three times the angle. Find the measurement of the angle and its supplement.**

$$x + (3x - 16) = 180$$
$$4x - 16 + 16 = 180 + 16$$
$$4x = 196$$
$$\left(\frac{1}{4}\right)4x = \left(\frac{1}{4}\right)196$$
$$x = 49$$

Angle $= 49°$

Supplement $= 3(49)° - 16° = 131°$

7. **The measurement of the complement of an angle exceeds the measure of the angle by 25%. Find the measurement of the angle and its complement.**

$$x + \left(x + \frac{1}{4}x\right) = 90$$
$$x + \frac{5}{4}x = 90$$
$$\frac{9}{4}x = 90$$
$$\left(\frac{4}{9}\right)\frac{9}{4}x = \left(\frac{4}{9}\right)90$$
$$x = 40$$

Angle $= 40°$

Complement $= \frac{5}{4}(40)° = 50°$

8. **The ratio of the measurement of an angle to its complement is $1:2$. Find the measruement of the angle and its complement.**

$$x + 2x = 90$$
$$3x = 90$$
$$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)90$$
$$x = 30$$

Angle $= 30°$

Complement $= 2(30)° = 60°$

9. **The ratio of the measurement of an angle to its supplement is $3:5$. Find the measurement of the angle and its supplement.**

$$3x + 5x = 180$$
$$8x = 180$$
$$\left(\frac{1}{8}\right)8x = \left(\frac{1}{8}\right)180$$
$$x = 22.5$$

Angle $= 3(22.5)° = 67.5°$

Supplement $= 5(22.5)° = 112.5°$

10. **Let x represent the measurement of an acute angle in degrees. The ratio of the complement of x to the supplement of x is $2:5$. Guess and check to determine the value of x. Explain why your answer is correct.**

Solutions will vary; $x = 30°$.

The complement of $30°$ ***is*** $60°$. ***The supplement of*** $30°$ ***is*** $150°$. ***The ratio of*** 60 ***to*** 150 ***is equivalent to*** $2:5$.