Lesson 2: Solving for Unknown Angles Using Equations

Student Outcomes

- Students solve for unknown angles in word problems and in diagrams involving complementary, supplementary, vertical, and adjacent angles.
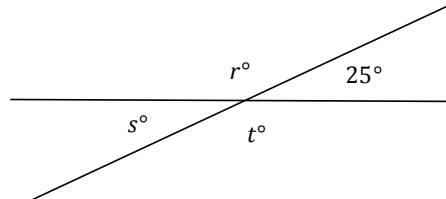
Classwork

Opening Exercise (5 minutes)

Opening Exercise

Two lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Find the values of r, s, and t.

The two intersecting lines form two pairs of vertical angles; $s = 25$, and $r° = t°$. Angles $s°$ and $r°$ are angles on a line and sum to $180°$.

$$s = 25$$
$$r + 25 = 180$$
$$r + 25 - 25 = 180 - 25$$
$$r = 155,$$
$$t = 155$$

MP.6

In the following examples and exercises, students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Model the good habit of always stating the geometric reason when you use one. This is a requirement in high school geometry.

Scaffolding:

Students may benefit from repeated practice drawing angle diagrams from verbal descriptions. For example, tell them "Draw a diagram of two supplementary angles, where one has a measure of 37°." Students struggling to organize their solution to a problem may benefit from the five-part process of the Exit Ticket in Lesson 1, including writing an equation, explaining the connection between the equation and the situation, and assessing whether an answer is reasonable. This builds conceptual understanding.

Example 1 (4 minutes)

Example 1

Two lines meet at a point that is also the endpoint of a ray. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of p and r.

The angle $r°$ is vertically opposite from and equal to the sum of the angles with measurements $28°$ and $16°$, or a sum of $44°$. Angles $r°$ and $p°$ are angles on a line and sum to $180°$.

$$r = 28 + 16 \qquad \text{Vert. } \angle s$$
$$r = 44$$
$$p + (44) = 180 \qquad \angle s \text{ on a line}$$
$$p + 44 - 44 = 180 - 44$$
$$p = 136$$
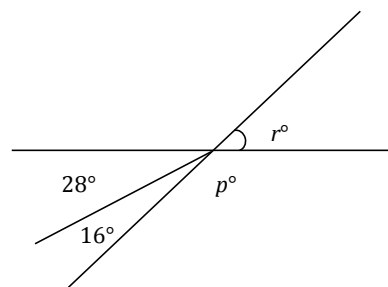
Take the opportunity to distinguish the correct usage of *supplementary* versus *angles on a line* in this example. Remind students that *supplementary* should be used in reference to two angles, whereas *angles on a line* can be used for two *or more* angles.

Exercise 1 (4 minutes)

> **Exercise 1**
>
> **Three lines meet at a point. In a complete sentence, describe the relevant angle relationship in the diagram. Set up and solve an equation to find the value of a.**
>
> *The two $a°$ angles and the angle $144°$ are angles on a line and sum to $180°$.*
>
> $$2a + 144 = 180 \qquad ∠s \text{ on a line}$$
> $$2a + 144 - 144 = 180 - 144$$
> $$2a = 36$$
> $$a = 18$$

Example 2 (4 minutes)

Encourage students to label diagrams as needed to facilitate their solutions. In this example, the label $y°$ is added to the diagram to show the relationship of $z°$ with $19°$. This addition allows for methodical progress toward the solution.

> **Example 2**
>
> **Three lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of z.**
>
> **Let $y°$ be the angle vertically opposite and equal in measurement to $19°$.**
> **The angles $z°$ and $y°$ are complementary and sum to $90°$.**
>
> $$z + y = 90 \qquad \text{Complementary } ∠s$$
> $$z + 19 = 90$$
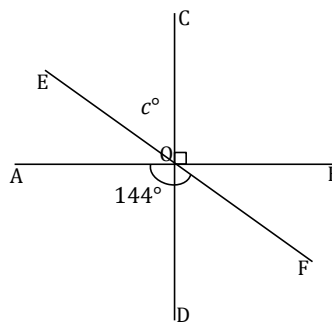> $$z + 19 - 19 = 90 - 19$$
> $$z = 71$$

Exercise 2 (4 minutes)

Exercise 2

Three lines meet at a point; $\angle AOF = 144°$. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to determine the value of c.

$\angle EOB$, formed by adjacent angles $\angle EOC$ and $\angle COB$, is vertical to and equal in measurement to $\angle AOF$.
The measurement of $\angle EOB$ is $c° + 90°$ ($\angle s$ add).

$$c + 90 = 144 \qquad \text{Vert. } \angle s$$
$$c + 90 - 90 = 144 - 90$$
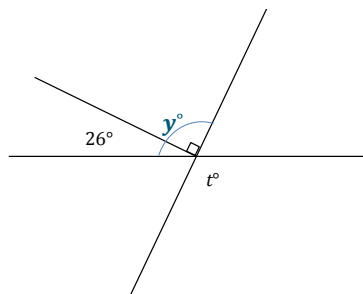$$c = 54$$

Example 3 (4 minutes)

Example 3

Two lines meet at a point that is also the endpoint of a ray. The ray is perpendicular to one of the lines as shown. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of t.

The measurement of the angle formed by adjacent angles of $26°$ and $90°$ is the sum of the adjacent angles. This angle is vertically opposite and equal in measurement to the angle $t°$.

Let $y°$ be the measure of the indicated angle.

$$y = 116 \qquad \angle s \text{ add}$$
$$t = (y) \qquad \text{Vert. } \angle s$$
$$t = 116$$

Exercise 3 (4 minutes)

Exercise 3

Two lines meet at a point that is also the endpoint of a ray. The ray is perpendicular to one of the lines as shown. In a complete sentence, describe the relevant angle relationships in the diagram. You may add labels to the diagram to help with your description of the angle relationship. Set up and solve an equation to find the value of v.

One possible response: Let $x°$ be the angle vertically opposite and equal in measurement to $46°$. The angles $x°$ and $v°$ are adjacent angles, and the angle they form together is equal to the sum of their measurements.

$$x = 46 \qquad \text{Vert. } \angle s$$
$$v = 90 + 46 \qquad \angle s \text{ add}$$
$$v = 136$$
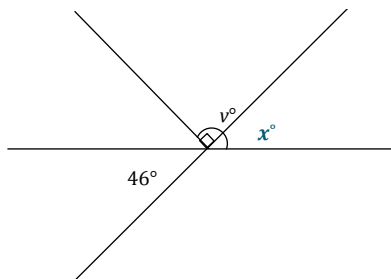
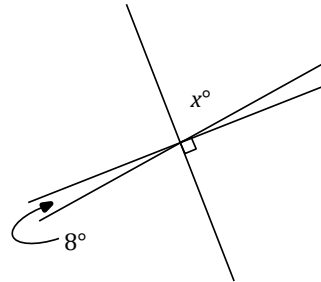## Example 4 (4 minutes)

Example 4

Three lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Is your answer reasonable? Explain how you know.

The angle $x°$ is vertically opposite from the angle formed by the right angle that contains and shares a common side with an 8° angle.

$$x = 90 - 8 \qquad \angle s \text{ add and vert. } \angle s$$
$$x = 82$$

The answer is reasonable because the angle marked by $x°$ is close to appearing as a right angle.

Exercise 4 (4 minutes)

Exercise 4

Two lines meet at a point that is also the endpoint of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of $\angle AOB$ and $\angle BOC$.

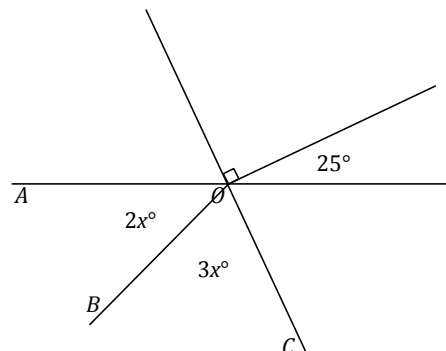*$\angle AOC$ is vertically opposite from the angle formed by adjacent angles 90° and 25°.*

$$2x + 3x = 90 + 25 \qquad \angle s \text{ add and vert. } \angle s$$
$$5x = 115$$
$$x = 23$$

$$\angle AOC = 2(23)° = 46°$$
$$\angle BOC = 3(23)° = 69°$$

Exercise 5 (4 minutes)

Exercise 5

a.   In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of $\angle AOB$ and $\angle BOC$.

$\angle AOB$ and $\angle BOC$ are complementary and sum to $90°$.

$$5x + (2x + 20) = 90 \qquad \text{complementary } \angle s$$
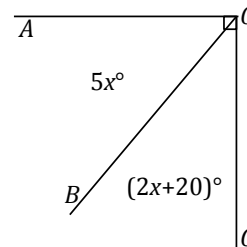$$7x + 20 = 90$$
$$7x + 20 - 20 = 90 - 20$$
$$7x = 70$$
$$x = 10$$

$\angle AOB = 5(10)° = 50°$
$\angle BOC = 2(10)° + 20° = 40°$

b.   Katrina was solving the problem above and wrote the equation $7x + 20 = 90$. Then, she rewrote this as $7x + 20 = 70 + 20$. Why did she rewrite the equation in this way? How does this help her to find the value of x?

She grouped the quantity on the right-hand side of the equation similarly to that of the left-hand side. This way, it is clear that the quantity $7x$ on the left-hand side must be equal to the quantity 70 on the right-hand side.

MP.7

Closing (1 minute)

▪ In every unknown angle problem, it is important to identify the angle relationship(s) correctly in order to set up an equation that yields the unknown value.

▪ Check your answer by substituting and/or measuring to be sure it is correct.

Lesson Summary

▪ To solve an unknown angle problem, identify the angle relationship(s) first to set up an equation that will yield the unknown value.

▪ Angles on a line and supplementary angles are not the same relationship. *Supplementary* angles are two angles whose angle measures sum to $180°$ whereas *angles on a line* are two or more adjacent angles whose angle measures sum to $180°$.
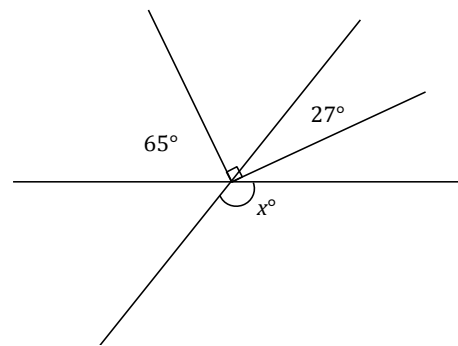

Exit Ticket (3 minutes)

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Exit Ticket

Two lines meet at a point that is also the vertex of an angle. Set up and solve an equation to find the value of x. Explain why your answer is reasonable.

Exit Ticket Sample Solutions

Two lines meet at a point that is also the vertex of an angle. Set up and solve an equation to find the value of x. Explain why your answer is reasonable.

$$65 + (90 - 27) = x$$
$$x = 128$$

OR

$$y + 27 = 90$$
$$y + 27 - 27 = 90 - 27$$
$$y = 63$$

$$65 + y = x$$
$$65 + (63) = x$$
$$x = 128$$
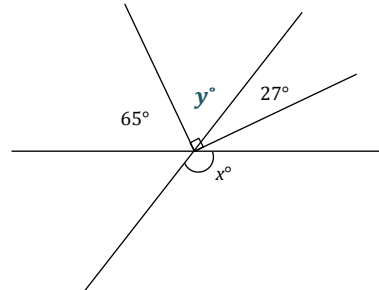
The answers seem reasonable because a rounded value of y as 60 and a rounded value of its adjacent angle 65 as 70 yields a sum of 130, which is close to the calculated answer.

Problem Set Sample Solutions

Note: Arcs indicating unknown angles begin to be dropped from the diagrams. It is necessary for students to determine the specific angle whose measure is being sought. Students should draw their own arcs.
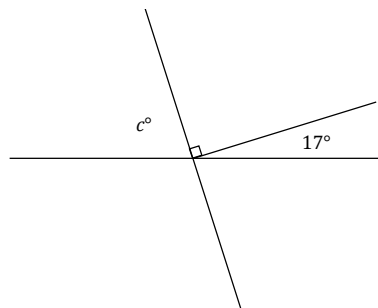
1. Two lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of c.

$$c + 90 + 17 = 180 \qquad \text{∠s on a line}$$
$$c + 107 = 180$$
$$c + 107 - 107 = 180 - 107$$
$$c = 73$$

Scaffolding:

Students struggling to organize their solution may benefit from prompts such as the following: Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

Scaffolded solutions:

a. *Use the equation above.*

b. *The angle marked $c°$, the right angle, and the angle with measurement $17°$ are angles on a line, and their measurements sum to $180°$.*

c. *Use the solution above. The answer seems reasonable because it looks like it has a measurement a little less than a $90°$ angle.*

2. Two lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of a. Explain why your answer is reasonable.
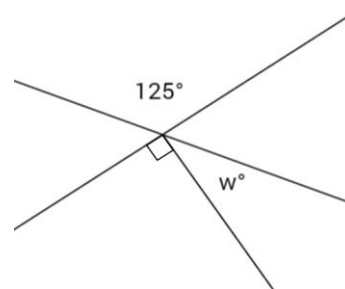
$$a + 33 = 49 \qquad \angle s \text{ add and vert. } \angle s$$
$$a + 33 - 33 = 49 - 33$$
$$a = 16$$

The answers seem reasonable because a rounded value of a as 20 and a rounded value of its adjacent angle 33 as 30 yields a sum of 50, which is close to the rounded value of the measurement of the vertical angle.
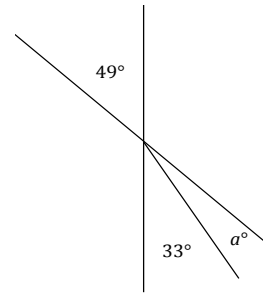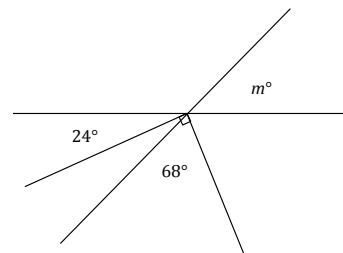
3. Two lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of w.

$$w + 90 = 125 \qquad \angle s \text{ add and vert. } \angle s$$
$$w + 90 - 90 = 125 - 90$$
$$w = 35$$
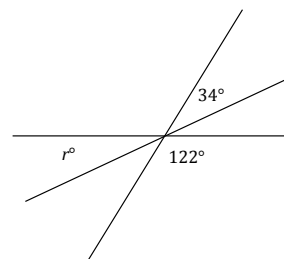
4. Two lines meet at a point that is also the vertex of an angle. Set up and solve an equation to find the value of m.

$$(90 - 68) + 24 = m \qquad \angle s \text{ add and vert. } \angle s$$
$$m = 46$$

5. Three lines meet at a point. Set up and solve an equation to find the value of r.

$$r + 122 + 34 = 180 \qquad \angle s \text{ on a line and vert. } \angle s$$
$$r + 156 = 180$$
$$r + 156 - 156 = 180 - 156$$
$$r = 24$$

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6. Three lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of each variable in the diagram.
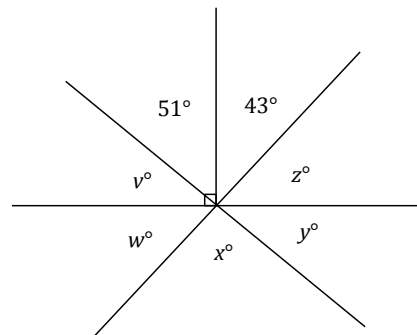
$v = 90 - 51$ *Complementary* $\angle s$
$v = 39$

$w + 39 + 51 + 43 = 180$ $\angle s$ *on a line*
$w + 133 = 180$
$w + 133 - 133 = 180 - 133$
$w = 47$

$x = 51 + 43$ *Vert.* $\angle s$
$x = 94$

$y = 39$ *Vert.* $\angle s$

$z = 47$ *Vert.* $\angle s$

7. Set up and solve an equation to find the value of x. Find the measurement of $\angle AOB$ and of $\angle BOC$.
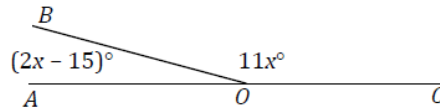
$(2x - 15) + 11x = 180$ *Supplementary* $\angle s$
$13x - 15 = 180$
$13x - 15 + 15 = 180 + 15$
$13x = 195$
$x = 15$

The measurement of $\angle AOB$: $2(15)° - 15° = 15°$

The measurement of $\angle BOC$: $11(15)° = 165°$

Scaffolded solutions:

a. Use the equation above.

b. The marked angles are angles on a line, and their measurements sum to $180°$.

c. Once 15 is substituted for x, then the measurement of $\angle AOB$ is $15°$ and the measurement of $\angle BOC$ is $165°$. These answers seem reasonable since $\angle AOB$ is acute and $\angle BOC$ is obtuse.

> *Scaffolding:*
>
> Students struggling to organize their solution may benefit from prompts such as the following:
>
> - Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

8. Set up and solve an equation to find the value of x. Find the measurement of $\angle AOB$ and of $\angle BOC$.

$x + 8 + 2x = 90$ *Complementary* $\angle s$
$3x + 8 = 90$
$3x + 8 - 8 = 90 - 8$
$3x = 82$
$x = 27\frac{1}{3}$

The measurement of $\angle AOB$: $\left(27\frac{1}{3}\right)° + 8° = 35\frac{1}{3}°$

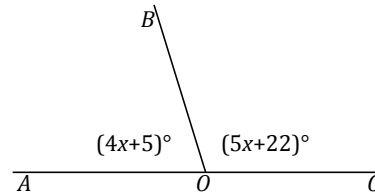*The measurement of* $\angle BOC$: $2\left(27\frac{1}{3}\right)° = 54\frac{2}{3}°$
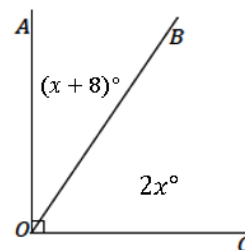
9.   Set up and solve an equation to find the value of x. Find the measurement of $\angle AOB$ and of $\angle BOC$.

$$4x + 5 + 5x + 22 = 180 \qquad \angle s \text{ on a line}$$
$$9x + 27 = 180$$
$$9x + 27 - 27 = 180 - 27$$
$$9x = 153$$
$$x = 17$$

The measurement of $\angle AOB$: $4(17)° + 5° = 73°$

The measurement of $\angle BOC$: $5(17)° + 22° = 107°$

10. Write a verbal problem that models the following diagram. Then, solve for the two angles.

One possible response: Two angles are supplementary. The measurement of one angle is five times the measurement of the other. Find the measurements of both angles.

$$10x + 2x = 180 \qquad Supplementary \angle s$$
$$12x = 180$$
$$x = 15$$

The measurement of Angle 1: $10(15)° = 150°$

The measurement of Angle 2: $2(15)° = 30°$