



Lesson 3: Solving for Unknown Angles Using Equations

Student Outcomes

- Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.

Classwork

Opening Exercise (5 minutes)

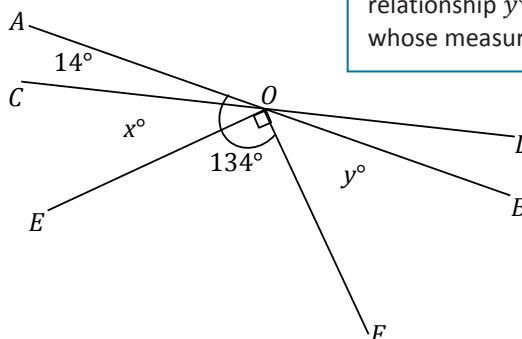
Opening Exercise

Two lines meet at a point that is also a vertex of an angle; the measurement of $\angle AOF$ is 134° . Set up and solve an equation to find the values of x and y . Are your answers reasonable? How do you know?

$$\begin{aligned}
 14 + x + 90 &= 134 && \angle s \text{ add} \\
 x + 104 &= 134 \\
 x + 104 - 104 &= 134 - 104 \\
 x &= 30
 \end{aligned}$$

$$\begin{aligned}
 y + 134 &= 180 && \angle s \text{ on a line} \\
 y + 134 - 134 &= 180 - 134 \\
 y &= 46
 \end{aligned}$$

The answers are reasonable because the angle marked y° appears to be approximately half the measurement of a right angle, and the angle marked x° appears to be approximately double in measurement of $\angle AOC$.



Scaffolding:

Encourage students to redraw parts of the diagram to emphasize relationships. For example, line AB and ray OE could be redrawn to see the relationship y° and the angle whose measure is 134° .

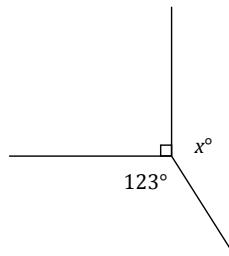
In the following examples and exercises, students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Encourage students to note the appropriate angle fact abbreviation for any step that depends on an angle relationship.

Example 1 (4 minutes)

Example 1

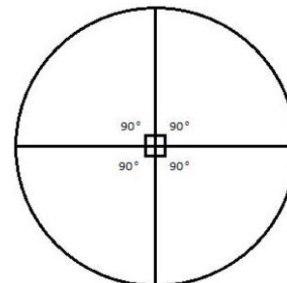
Set up and solve an equation to find the value of x .

$$\begin{aligned}
 x + 90 + 123 &= 360 && \angle s \text{ at a point} \\
 x + 213 &= 360 \\
 x + 213 - 213 &= 360 - 213 \\
 x &= 147
 \end{aligned}$$



Scaffolding:

Remind students that a full rotation or turn through a circle is 360° .



A circular protractor may help to demonstrate this.

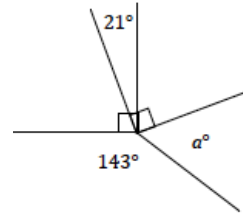
Exercise 1 (4 minutes)

Exercise 1

Five rays meet at a common endpoint. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of a .

The sum of angles at a point is 360° .

$$\begin{aligned} 90 + (90 - 21) + a + 143 &= 360 && \angle\text{s at a point} \\ 302 + a &= 360 \\ 302 - 302 + a &= 360 - 302 \\ a &= 58 \end{aligned}$$



Example 2 (4 minutes)

Example 2

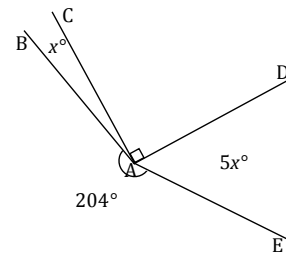
Four rays meet at a common endpoint. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x . Find the measurements of $\angle BAC$ and $\angle DAE$.

The sum of the degree measurements of $\angle BAC$, $\angle CAD$, $\angle DAE$ and the arc that measures 204° is 360° .

$$\begin{aligned} x + 90 + 5x + 204 &= 360 && \angle\text{s at a point} \\ 6x + 294 &= 360 \\ 6x + 294 - 294 &= 360 - 294 \\ 6x &= 66 \\ \left(\frac{1}{6}\right) 6x &= \left(\frac{1}{6}\right) 66 \\ x &= 11 \end{aligned}$$

The measurement of $\angle BAC$: 11°

The measurement of $\angle DAE$: $5(11)^\circ = 55^\circ$



Exercise 2 (4 minutes)

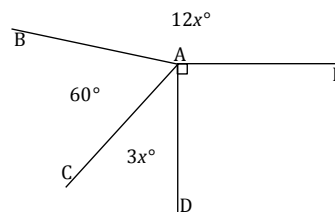
Exercise 2

Four rays meet at a common endpoint. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x . Find the measurement of $\angle CAD$.

$\angle BAC$, $\angle CAD$, $\angle DAE$, and $\angle EAB$ are angles at a point and sum to 360° .

$$\begin{aligned} 3x + 60 + 12x + 90 &= 360 && \angle\text{s at a point} \\ 15x + 150 &= 360 \\ 15x + 150 - 150 &= 360 - 150 \\ 15x &= 210 \\ \left(\frac{1}{15}\right) 15x &= \left(\frac{1}{15}\right) 210 \\ x &= 14 \end{aligned}$$

The measurement of $\angle CAD$: $3(14)^\circ = 42^\circ$



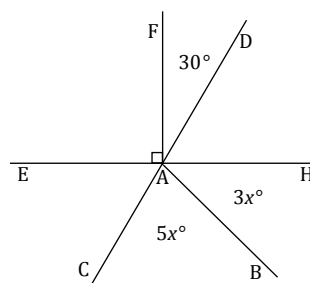
Example 3 (4 minutes)

Example 3

Two lines meet at a point that is also the endpoint of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x . Find the measurements of $\angle BAC$ and $\angle BAH$.

$\angle DAE$ is formed by adjacent angles $\angle EAF$ and $\angle FAD$; the measurement of $\angle DAE$ is equal to the sum of the measurements of the adjacent angles. This is also true for the measurement of $\angle CAH$, formed by adjacent angles $\angle CAB$ and $\angle BAH$. $\angle CAH$ is vertically opposite from and equal in measurement to $\angle DAE$.

$$\begin{aligned} 90 + 30 &= 120 && \angle DAE, \angle s \text{ add} \\ 5x + 3x &= 8x && \angle CAH, \angle s \text{ add} \\ \\ 8x &= 120 && \text{Vert. } \angle s \\ \left(\frac{1}{8}\right) 8x &= \left(\frac{1}{8}\right) 120 \\ x &= 15 \end{aligned}$$



The measurement of $\angle BAC$: $5(15)^\circ = 75^\circ$

The measurement of $\angle BAH$: $3(15)^\circ = 45^\circ$

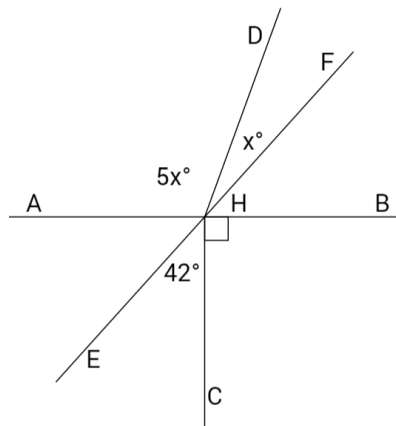
Exercise 3 (4 minutes)

Exercise 3

Lines AB and EF meet at a point which is also the endpoint of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x . Find the measurements of $\angle DHF$ and $\angle AHD$.

The measurement of $\angle AHF$, formed by adjacent angles $\angle AHD$ and $\angle DHF$, is equal to the sum of the measurements of the adjacent angles. This is also true for the measurement of $\angle EHB$, which is formed by adjacent angles $\angle EHC$ and $\angle CHB$. $\angle AHF$ is vertically opposite from and equal in measurement to $\angle EHB$.

$$\begin{aligned} 5x + x &= 6x && \angle AHF, \angle s \text{ add} \\ 42 + 90 &= 132 && \angle EHB, \angle s \text{ add} \\ \\ 6x &= 132 && \text{Vert. } \angle s \\ \left(\frac{1}{6}\right) 6x &= \left(\frac{1}{6}\right) 132 \\ x &= 22 \end{aligned}$$



The measurement of $\angle DHF$: 22°

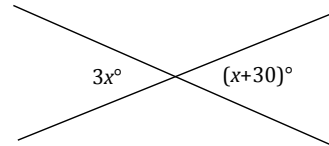
The measurement of $\angle AHD$: $5(22)^\circ = 110^\circ$

The following examples are designed to highlight MP.7 by helping students to see the connection between an angle diagram and the equation used to model it. Solving equations with variables on both sides is a topic in Grade 8. Teachers may choose to show the solution method if they so choose.

Example 4 (6 minutes)

Example 4

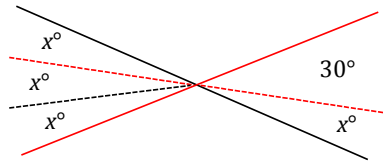
Two lines meet at a point. Set up and solve an equation to find the value of x . Find the measurement of one of the vertical angles.



Students use information in the figure and a protractor to solve for x .

- i) Students measure a 30° angle as shown; the remaining portion of the angle must be x° (\angle s add).
- ii) Students can use their protractor to find the measurement of x° and use this measurement to partition the other angle in the vertical pair.

As a check, students should substitute the measured x value into each expression and evaluate; each angle of the vertical pair should equal the other. Students can also use their protractor to measure each angle of the vertical angle pair.



MP.7

With a modified figure, students can write an algebraic equation that they have the skills to solve.

$$2x = 30 \quad \text{Vert. } \angle\text{s}$$

$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)30$$

$$x = 15$$

Measurement of each angle in the vertical pair: $3(15)^\circ = 45^\circ$

Extension: The algebra steps above are particularly helpful as a stepping-stone in demonstrating how to solve the equation that takes care of the problem in one step as follows:

$$3x = x + 30 \quad \text{Vert. } \angle\text{s}$$

$$3x - x = x - x + 30$$

$$2x = 30$$

$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)30$$

$$x = 15$$

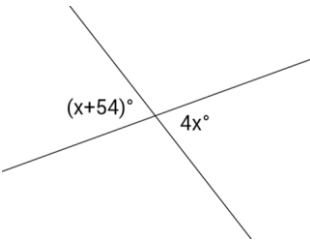
Measurement of each angle in the vertical pair: $3(15)^\circ = 45^\circ$

Students understand the first line of this solution because of their knowledge of vertical angles. In fact, the only line they are not familiar with is the second line of the solution, which is a skill that they learn in Grade 8. Showing students this solution is simply a preview.

Exercise 4 (4 minutes)

Exercise 4

Set up and solve an equation to find the value of x . Find the measurement of one of the vertical angles.



Students use information in the figure and a protractor to solve for x .

- i) Students measure a 54° angle as shown; the remaining portion of the angle must be x (\angle s add).
- ii) Students can use their protractors to find the measurement of x and use this measurement to partition the other angle in the vertical pair.

Students should perform a check as in Example 4 before solving an equation that matches the modified figure.

$54 = 3x$ *Vert. \angle s*

$\left(\frac{1}{3}\right) 54 = \left(\frac{1}{3}\right) 3x$

$x = 18$

Measurement of each vertical angle: $4(18)^\circ = 72^\circ$

Extension:

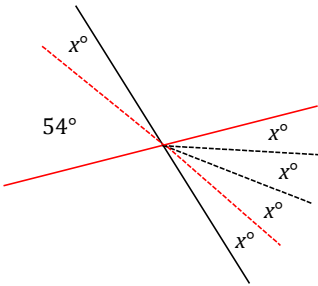
$x + 54 = 4x$ *Vert. \angle s*

$x - x + 54 = 4x - x$

$54 = 3x$

$\left(\frac{1}{3}\right) 54 = \left(\frac{1}{3}\right) 3x$

$x = 18$



Closing (1 minute)

- In every unknown angle problem, it is important to identify the angle relationship(s) correctly in order to set up an equation that yields the unknown value.
- Check your answer by substituting and/or measuring to be sure it is correct.

Lesson Summary**Steps to Solving for Unknown Angles**

- Identify the angle relationship(s).
- Set up an equation that will yield the unknown value.
- Solve the equation for the unknown value.
- Substitute the answer to determine the angle(s).
- Check and verify your answer by measuring the angle with a protractor.

Exit Ticket (5 minutes)

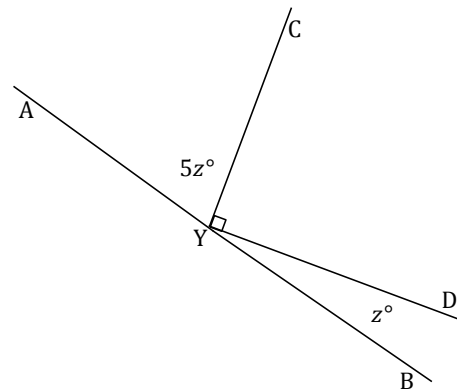
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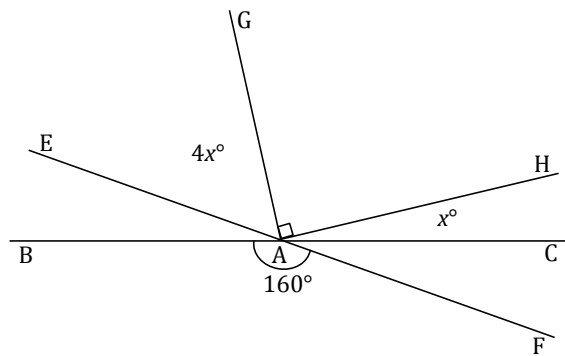
Lesson 3: Solving for Unknown Angles Using Equations

Exit Ticket

- Two rays have a common endpoint on a line. Set up and solve an equation to find the value of z . Find the measurements of $\angle AYC$ and $\angle DYB$.



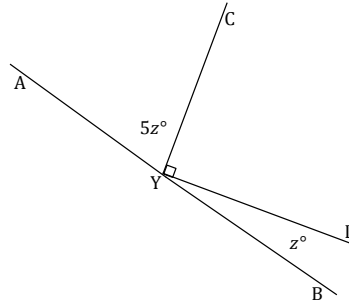
- Two lines meet at a point that is also the vertex of an angle. Set up and solve an equation to find the value of x . Find the measurements of $\angle CAH$ and $\angle EAG$.



Exit Ticket Sample Solutions

1. Two rays have a common endpoint on a line. Set up and solve an equation to find the value of z . Find the measurements of $\angle AYC$ and $\angle DYB$.

$$\begin{aligned}
 5z + 90 + z &= 180 && \angle s \text{ on a line} \\
 6z + 90 &= 180 \\
 6z + 90 - 90 &= 180 - 90 \\
 6z &= 90 \\
 \left(\frac{1}{6}\right)6z &= \left(\frac{1}{6}\right)90 \\
 z &= 15
 \end{aligned}$$



Scaffolding:
 Students struggling to organize their solution may benefit from prompts such as the following: Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

The measurement of $\angle AYC$: $5(15)^\circ = 75^\circ$

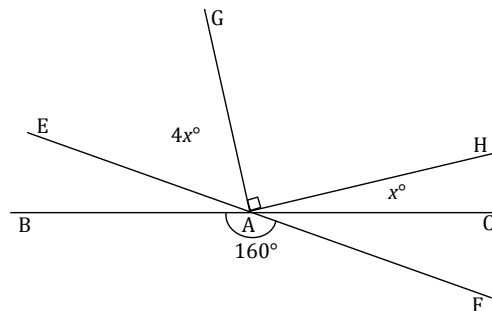
The measurement of $\angle DYB$: 15°

Scaffolded solutions:

- Use the equation above.
- The angle marked z° , the right angle, and the angle with measurement $5z^\circ$ are angles on a line. Their measurements sum to 180° .
- The answers seem reasonable because once 15 is substituted in for z , the measurement of $\angle AYC$ is 75° , which is slightly smaller than a right angle, and the measurement of $\angle DYB$ is 15° , which is an acute angle.

2. Two lines meet at a point that is also the vertex of an angle. Set up and solve an equation to find the value of x . Find the measurements of $\angle CAH$ and $\angle EAG$.

$$\begin{aligned}
 4x + 90 + x &= 160 && \text{vert. } \angle s \\
 5x + 90 &= 160 \\
 5x + 90 - 90 &= 160 - 90 \\
 5x &= 70 \\
 \left(\frac{1}{5}\right)5x &= \left(\frac{1}{5}\right)70 \\
 x &= 14
 \end{aligned}$$



The measurement of $\angle CAH$: 14°

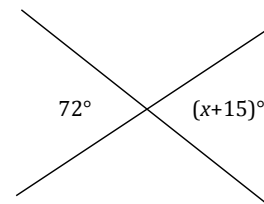
The measurement of $\angle EAG$: $4(14)^\circ = 56^\circ$

Problem Set Sample Solutions

Set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Add labels to diagrams as needed to facilitate their solutions. List the appropriate angle fact abbreviation for any step that depends on an angle relationship.

1. Two lines meet at a point. Set up and solve an equation to find the value of x .

$$\begin{aligned}
 x + 15 &= 72 && \text{Vert. } \angle s \\
 x + 15 - 15 &= 72 - 15 \\
 x &= 57
 \end{aligned}$$

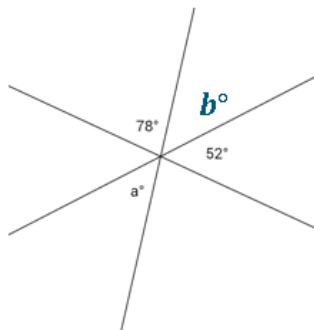


2. Three lines meet at a point. Set up and solve an equation to find the value of a . Is your answer reasonable? Explain how you know.

$$\begin{aligned} \text{Let } b &= a. && \text{Vert. } \angle\text{s} \\ 78 + b + 52 &= 180 && \angle\text{s on a line} \\ b + 130 &= 180 \\ b + 130 - 130 &= 180 - 130 \\ b &= 50 \end{aligned}$$

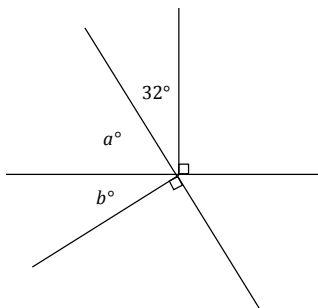
Since $b = a$, $a = 50$.

The answer seems reasonable since it is similar in magnitude to the 52° angle.



3. Two lines meet at a point that is also the endpoint of two rays. Set up and solve an equation to find the values of a and b .

$$\begin{aligned} a + 32 + 90 &= 180 && \angle\text{s on a line} \\ a + 122 &= 180 \\ a + 122 - 122 &= 180 - 122 \\ a &= 58 \\ \\ a + b + 90 &= 180 && \angle\text{s on a line} \\ 58 + b + 90 &= 180 \\ b + 148 &= 180 \\ b + 148 - 148 &= 180 - 148 \\ b &= 32 \end{aligned}$$



Scaffolding:

Students struggling to organize their solution may benefit from prompts such as the following:

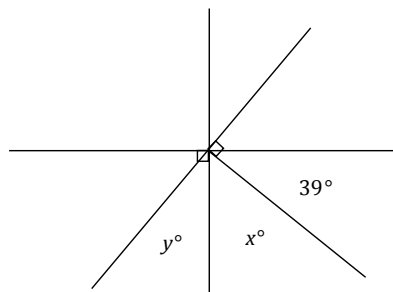
- Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

Scaffolded solutions:

- a. Use the equation above.
- b. The angle marked a° , the angle with measurement 32° , and the right angle are angles on a line. Their measurements sum to 180° .
- c. The answers seem reasonable because once the values of a and b are substituted, it appears that the two angles (a° and b°) form a right angle. We know those two angles should form a right angle because the angle adjacent to it is a right angle.

4. Three lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the values of x and y .

$$\begin{aligned} x + 39 + 90 &= 180 && \angle\text{s on a line} \\ x + 129 &= 180 \\ x + 129 - 129 &= 180 - 129 \\ x &= 51 \\ \\ y + x + 90 &= 180 && \angle\text{s on a line} \\ y + 51 + 90 &= 180 \\ y + 141 &= 180 \\ y + 141 - 141 &= 180 - 141 \\ y &= 39 \end{aligned}$$

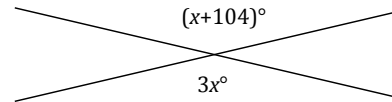


5. Two lines meet at a point. Find the measurement of one of the vertical angles. Is your answer reasonable? Explain how you know.

$$2x = 104 \quad \text{vert. } \angle s$$

$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)104$$

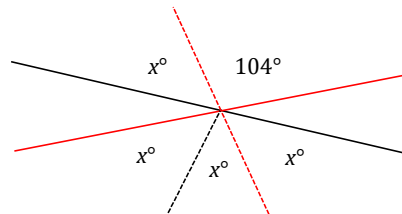
$$x = 52$$



Measurement of each vertical angle: $3(52)^\circ = 156^\circ$

The answer seems reasonable because a rounded value of 50 would make the numeric value of each expression 150 and 154, which are reasonably close for a check.

A solution can include a modified diagram, as shown, and the supporting algebra work.



Solutions may also include the full equation and solution:

$$3x = x + 104 \quad \text{Vert. } \angle s$$

$$3x - x = x - x + 104$$

$$2x = 104$$

$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)104$$

$$x = 52$$

6. Three lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of y .

Let x° and z° be the measurements of the indicated angles.

$$x + 15 = 90 \quad \text{Vert. } \angle s$$

$$x + 15 - 15 = 90 - 15$$

$$x = 75$$

$$x + z = 90 \quad \text{Complementary } \angle s$$

$$75 + z = 90$$

$$75 - 75 + z = 90 - 75$$

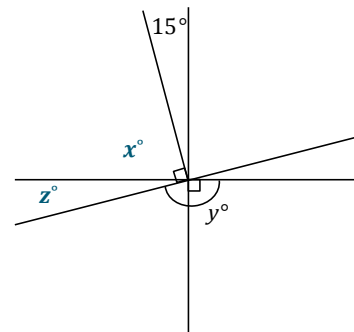
$$z = 15$$

$$z + y = 180 \quad \angle s \text{ on a line}$$

$$15 + y = 180$$

$$15 - 15 + y = 180 - 15$$

$$y = 165$$



7. Three adjacent angles are at a point. The second angle is 20° more than the first, and the third angle is 20° more than the second angle.

- a. Find the measurements of all three angles.

$$\begin{aligned} x + (x + 20) + (x + 20 + 20) &= 360 && \angle s \text{ at a point} \\ 3x + 60 &= 360 \\ 3x + 60 - 60 &= 360 - 60 \\ 3x &= 300 \\ \left(\frac{1}{3}\right) 3x &= \left(\frac{1}{3}\right) 300 \\ x &= 100 \end{aligned}$$

Angle 1: 100°

Angle 2: $100^\circ + 20^\circ = 120^\circ$

Angle 3: $100^\circ + 20^\circ + 20^\circ = 140^\circ$

- b. Compare the expressions you used for the three angles and their combined expression. Explain how they are equal and how they reveal different information about this situation.

By the commutative and associative laws, $x + (x + 20) + (x + 20 + 20)$ is equal to $(x + x + x) + (20 + 20 + 20)$, which is equal to $3x + 60$. The first expression, $x + (x + 20) + (x + 20 + 20)$, shows the sum of three unknown numbers, where the second is 20 more than the first, and the third is 20 more than the second. The expression $3x + 60$ shows the sum of three times an unknown number with 60.

8. Four adjacent angles are on a line. The measurements of the four angles are four consecutive even numbers. Determine the measurements of all four angles.

$$\begin{aligned} x + (x + 2) + (x + 4) + (x + 6) &= 180 && \angle s \text{ on a line} \\ 4x + 12 &= 180 \\ 4x + 12 - 12 &= 180 - 12 \\ 4x &= 168 \\ \left(\frac{1}{4}\right) 4x &= \left(\frac{1}{4}\right) 168 \\ x &= 42 \end{aligned}$$

The four angle measures are 42° , 44° , 46° , and 48° .

9. Three adjacent angles are at a point. The ratio of the measurement of the second angle to the measurement of the first angle is 4:3. The ratio of the measurement of the third angle to the measurement of the second angle is 5:4. Determine the measurements of all three angles.

Let the smallest measure of the three angles be $3x^\circ$. Then, the measure of the second angle is $4x^\circ$, and the measure of the third angle is $5x^\circ$.

$$\begin{aligned} 3x + 4x + 5x &= 360 && \angle s \text{ at a point} \\ 12x &= 360 \\ \left(\frac{1}{12}\right) 12x &= \left(\frac{1}{12}\right) 360 \\ x &= 30 \end{aligned}$$

Angle 1: $3(30)^\circ = 90^\circ$

Angle 2: $4(30)^\circ = 120^\circ$

Angle 3: $5(30)^\circ = 150^\circ$

Scaffolding:

Teachers may need to review the term *consecutive* for students to successfully complete Problem Set 8.

MP.2
&
MP.7

10. Four lines meet at a point. Solve for x and y in the following diagram.

$$2x + 18 + 90 = 180 \quad \angle\text{s on a line}$$

$$2x + 108 = 180$$

$$2x + 108 - 108 = 180 - 108$$

$$2x = 72$$

$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)72$$

$$x = 36$$

$$2x = 3y \quad \text{Vert. } \angle\text{s}$$

$$2(36) = 3y$$

$$72 = 3y$$

$$\left(\frac{1}{3}\right)72 = \left(\frac{1}{3}\right)3y$$

$$y = 24$$

