Lesson 4: Solving for Unknown Angles Using Equations

Student Outcomes

- Students solve for unknown angles in word problems and in diagrams involving all learned angle facts.

Classwork

Opening Exercise (5 minutes)

> **Opening Exercise**
>
> The complement of an angle is four times the measurement of the angle. Find the measurement of the angle and its complement.
>
> $$x + 4x = 90 \qquad \textit{Complementary } \angle s$$
> $$5x = 90$$
> $$\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)90$$
> $$x = 18$$
>
> *The measurement of the angle is $18°$.*
>
> *The measurement of the complement of the angle is $72°$.*

Scaffolding:

As in earlier lessons, tasks such as the Opening Exercise can be scaffolded into parts as follows:

- Explain the angle relationships in the diagram. Write the equation. Explain how the equation represents the diagram, including particular parts. Solve the equation. Interpret the solution, and determine if it is reasonable.

In the following examples and exercises, students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Encourage students to list the appropriate angle fact abbreviation for any step that depends on an angle relationship.

Example 1 (4 minutes)

Two options are provided here for Example 1. The second is more challenging than the first.
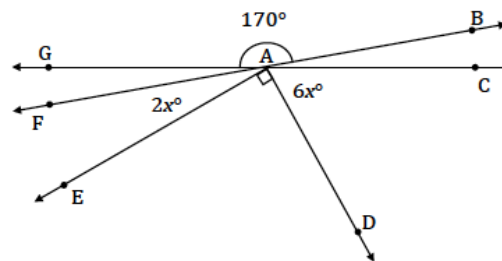
> **Example 1**
>
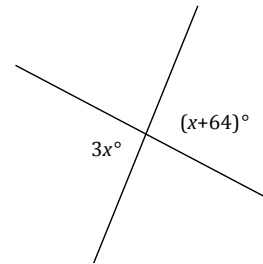> Find the measurements of $\angle FAE$ and $\angle CAD$.
>
> $$2x + 6x + 90 = 170 \qquad \textit{Vert. } \angle s$$
> $$8x + 90 = 170$$
> $$8x + 90 - 90 = 170 - 90$$
> $$8x = 80$$
> $$\left(\frac{1}{8}\right)8x = \left(\frac{1}{8}\right)80$$
> $$x = 10$$
>
>
>
> *The measurement of $\angle FAE$: $2(10)° = 20°$*
>
> *The measurement of $\angle CAD$: $6(10)° = 60°$*

Two lines meet at a point. List the relevant angle relationship in the diagram. Set up and solve an equation to find the value of x. Find the measurement of one of the vertical angles.

$(x+64)°$

$3x°$

Students use information in the figure and a protractor to solve for x.

    i)   Students measure a 64° angle as shown; the remaining portion of the angle must be $x°$ (∠s add).

    ii)   Students can use their protractors to find the measurement of $x°$ and use this measurement to partition the other angle in the vertical pair.

$x°$

$x°$

$x°$

$x°$

$64°$
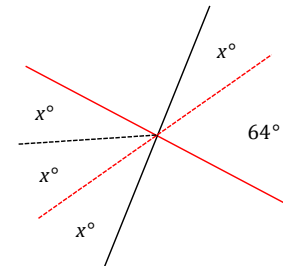
As a check, students should substitute the measured x value into each expression and evaluate; each angle of the vertical pair should be equal to the other. Students can also use their protractor to measure each angle of the vertical angle pair.

MP.7

With a modified figure, students can write an algebraic equation that they have the skills to solve.

$$2x = 64 \qquad\qquad \textit{Vert. ∠s}$$
$$\left(\tfrac{1}{2}\right)2x = \left(\tfrac{1}{2}\right)64$$
$$x = 32$$

Measurement of each angle in the vertical pair: $3(32)° = 96°$

Extension:

$$3x = x + 64 \qquad\qquad \textit{vert. ∠s}$$
$$3x - x = x - x + 64$$
$$2x = 64$$
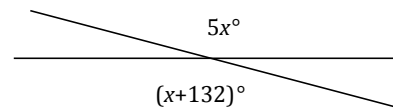$$\left(\tfrac{1}{2}\right)2x = \left(\tfrac{1}{2}\right)64$$
$$x = 32$$

Measurement of each angle in the vertical pair: $3(32)° = 96°$
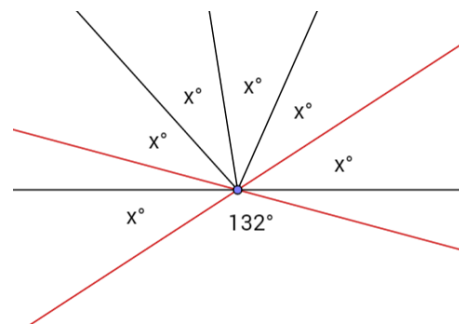
Exercise 1 (4 minutes)

Exercise 1

Set up and solve an equation to find the value of x. List the relevant angle relationship in the diagram. Find the measurement of one of the vertical angles.

$5x°$

$(x+132)°$

Students use information in the figure and a protractor to solve for x.

i) Measure a 132° angle as shown; the remaining portion of the original angle must be $x°$ (∠s add).

ii) Partition the other angle in the vertical pair into equal angles of $x°$.

Students should perform a check (as in Example 1) before solving an equation that matches the modified figure.

Extension:

$x°$ $x°$ $x°$ $x°$ $x°$ $x°$ 132°

$$4x = 132 \qquad\qquad \textit{vert. } \angle s$$
$$\left(\tfrac{1}{4}\right)4x = \left(\tfrac{1}{4}\right)132$$
$$x = 33$$

Measurement of each vertical angle: $5(33)° = 165°$

Note: Students can check their answers for any question by measuring each unknown angle with a protractor, as all diagrams are drawn to scale.

Example 2 (4 minutes)

Example 2

Three lines meet at a point. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of b.
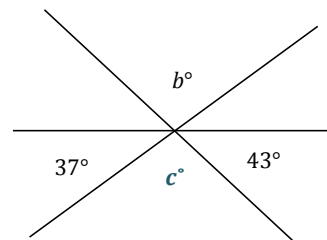
Let $b = c$.

$$c + 37 + 43 = 180 \qquad \textit{∠s on a line}$$
$$c + 80 = 180$$
$$c + 80 - 80 = 180 - 80$$
$$c = 100$$

$b°$

37° $c°$ 43°

Since $b = c$, $b = 100$.

Exercise 2 (4 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. List the appropriate angle fact abbreviation in the initial equation.

Exercise 2

Two lines meet at a point that is also the endpoint of two rays. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of b.

$$b + 95 = 90 + 80 \qquad \textit{Vert. } \angle s$$
$$b + 95 = 170$$
$$b + 95 - 95 = 170 - 95$$
$$b = 75$$

95°

$b°$ 80°

Example 3 (6 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationship in the question. In this case, suggest that students use the words *angle* and *supplement* as placeholders in their equations. Students can use a tape diagram to solve for the unknown angles.

Example 3

The measurement of an angle is $\dfrac{2}{3}$ the measurement of its supplement. Find the measurements of the angle and its supplement.

$$\textbf{angle} = \frac{2}{3}(\textbf{supplement})$$

$$\textbf{angle} = \frac{2}{3}(\textbf{180} - \textbf{angle})$$

Using a tape diagram:

angle

supplement

180

$5 \textit{ units} = 180$
$1 \textit{ unit} = 36$
$2 \textit{ units} = 72$
$3 \textit{ units} = 108$

The measurements of the two supplementary angles that satisfy these criteria are $72°$ and $108°$.

The tape diagram model is an ideal strategy for this question. If students are not familiar with the tape diagram model, use a Guess and Check table with them. Here is an example of such a table with two entries for guesses that did not result in a correct answer.

Guess	$\dfrac{2}{3}$(Guess)	Sum (should be 180°)
60	$\dfrac{2}{3}(60) = 40$	$60 + 40 = 100$; not the answer
90	$\dfrac{2}{3}(90) = 60$	$90 + 60 = 150$; not the answer

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Exercise 3 (5 minutes)

Students set up and solve an equation for the unknown angle based on the relevant angle relationship in the question. In this case, suggest that students use the words *angle* and *complement* as placeholders in their equations. Students can use a tape diagram to solve for the unknown angles.

Exercise 3

The measurement of an angle is $\frac{1}{4}$ the measurement of its complement. Find the measurements of the two complementary angles.

$$\textbf{angle} = \frac{1}{4}(\textbf{complement})$$

$$\textbf{angle} = \frac{1}{4}(90 - \textbf{angle})$$

Using a tape diagram:

angle

complement

90

5 *units* = 90

1 *unit* = 18

4 *units* = 72

The measurements of the two complementary angles that satisfy these criteria are 18° *and* 72°.

Example 4 (4 minutes)

Example 4

Three lines meet at a point that is also the endpoint of a ray. List the relevant angle relationships in the diagram. Set up and solve an equation to find the value of z.

Let $x°$ be the measurement of the indicated angle.

$$x + 90 + 29 = 180 \qquad \angle s \text{ on a line}$$
$$x + 119 = 180$$
$$x + 119 - 119 = 180 - 119$$
$$x = 61$$

$$z = x + 90 \qquad \angle s \text{ add}$$
$$z = 61 + 90$$
$$z = 151$$

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Exercise 4 (4 minutes)

Exercise 4

Two lines meet at a point that is also the vertex of an angle. Set up and solve an equation to find the value of x. Find the measurements of $\angle GAF$ and $\angle BAC$.

Let $y°$ be the measurement of the indicated angle.

$$y = 180 - (90 + 36) \qquad \text{∠s on a line}$$
$$y = 54$$

$$4x + y + 5x = 180 \qquad \text{∠s on a line}$$
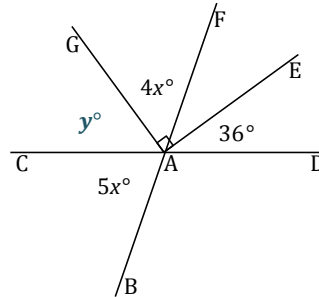$$4x + 54 + 5x = 180$$
$$9x + 54 = 180$$
$$9x + 54 - 54 = 180 - 54$$
$$9x = 126$$
$$\left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)126$$
$$x = 14$$

The measurement of $\angle GAF$: $4(14)° = 56°$

The measurement of $\angle BAC$: $5(14)° = 70°$

Closing (1 minute)

▪ In every unknown angle problem, it is important to identify the angle relationship(s) correctly in order to set up an equation that yields the unknown value.

▪ Check your answer by substituting and/or measuring to be sure it is correct.

Lesson Summary

Steps to Solving for Unknown Angles

- ▪ Identify the angle relationship(s).
- ▪ Set up an equation that will yield the unknown value.
- ▪ Solve the equation for the unknown value.
- ▪ Substitute the answer to determine the measurement of the angle(s).
- ▪ Check and verify your answer by measuring the angle with a protractor.
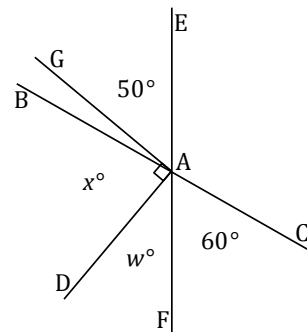
Exit Ticket (4 minutes)

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Lesson 4: Solving for Unknown Angles Using Equations

Exit Ticket

Lines BC and EF meet at A. Rays AG and AD form a right angle. Set up and solve an equation to find the values of x and w.

Exit Ticket Sample Solutions

Lines BC and EF meet at A. Rays AG and AD form a right angle. Set up and solve an equation to find the values of x and w.

$\angle BAE = 60$	Vert. $\angle s$
$\angle BAG = 10$	$\angle s$ add
$x + \angle BAG = 90$	Complementary $\angle s$
$x + 10 = 90$	
$x + 10 - 10 = 90 - 10$	
$x = 80$	
$x + w + 60 = 180$	$\angle s$ on a line
$80 + w + 60 = 180$	
$140 + w = 180$	
$140 + w - 140 = 180 - 140$	
$w = 40$	

Problem Set Sample Solutions

Set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Add labels to diagrams as needed to facilitate their solutions. List the appropriate angle fact abbreviation for any step that depends on an angle relationship.

Scaffolding:

Some students may need to use the corner of a piece of paper to confirm which rays form the right angle:
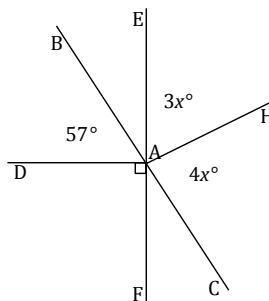$59 + d = 90$ or
$59 + d + c = 90$.

1. Four rays have a common endpoint on a line. Set up and solve an equation to find the value of c.

$59 + d = 90$	Complementary $\angle s$
$59 - 59 + d = 90 - 59$	
$d = 31$	
$d + c + 140 = 180$	$\angle s$ on a line
$31 + c + 140 = 180$	
$c + 171 = 180$	
$c + 171 - 171 = 180 - 171$	
$c = 9$	

2.   Lines BC and EF meet at A. Set up and solve an equation to find the value of x. Find the measurements of $\angle EAH$ and $\angle HAC$.

$$\angle BAE + 57 = 90 \qquad \textit{Complementary } \angle s$$
$$\angle BAE + 57 - 57 = 90 - 57$$
$$\angle BAE = 33$$

$$\angle BAE + 3x + 4x = 180 \qquad \textit{$\angle s$ on a line}$$
$$33 + 3x + 4x = 180$$
$$33 + 7x = 180$$
$$33 - 33 + 7x = 180 - 33$$
$$7x = 147$$
$$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)147$$
$$x = 21$$

Scaffolding:

Students struggling to organize their solution may benefit from prompts such as the following:

- Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

The measurement of $\angle EAH$: $3(21)° = 63°$

The measurement of $\angle HAC$: $4(21)° = 84°$

3.   Five rays share a common endpoint. Set up and solve an equation to find the value of x. Find the measurements of $\angle DAG$ and $\angle GAH$.

$$2x + 36.5 + 36.5 + 2x + 3x = 360 \qquad \textit{$\angle s$ at a point}$$
$$7x + 73 = 360$$
$$7x + 73 - 73 = 360 - 73$$
$$7x = 287$$
$$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)287$$
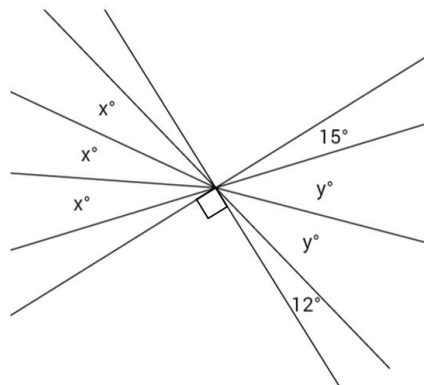$$x = 41$$

The measurement of $\angle EAF$: $2(41)° = 82°$

The measurement of $\angle GAH$: $3(41)° = 123°$
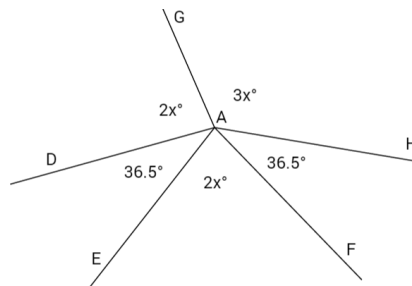
4.   Four lines meet at a point which is also the endpoint of three rays. Set up and solve an equation to find the values of x and y.

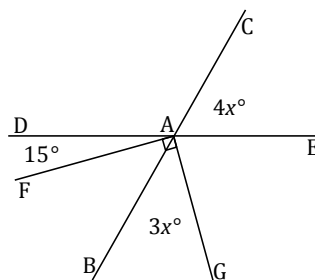$$2y + 12 + 15 + 90 = 180 \qquad \textit{$\angle s$ on a line}$$
$$2y + 117 = 180$$
$$2y + 117 - 117 = 180 - 117$$
$$2y = 63$$
$$\left(\frac{1}{2}\right)2y = \left(\frac{1}{2}\right)63$$
$$y = 31.5$$

$$3x = 2y \qquad \textit{Vert. $\angle s$}$$
$$3x = 2(31.5)$$
$$3x = 63$$
$$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)63$$
$$x = 21$$

5.   Two lines meet at a point that is also the vertex of a right angle. Set up and solve an equation to find the value of x. Find the measurements of $\angle CAE$ and $\angle BAG$.

$$\angle DAB = 4x \qquad\qquad \text{vert. } \angle s$$
$$\angle DAG = 90 + 15 = 105 \qquad \angle s \text{ add}$$
$$4x + 3x = 105$$
$$7x = 105 \qquad\qquad \angle s \text{ add}$$
$$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)105$$
$$x = 15$$

The measurement of $\angle CAE$: $4(15)° = 60°$

The measurement of $\angle BAG$: $3(15)° = 45°$

6.   Five angles are at a point. The measurement of each angle is one of five consecutive, positive whole numbers.

    a.   Determine the measurements of all five angles.

$$x + (x + 1) + (x + 2) + (x + 3) + (x + 4) = 360$$
$$5x + 10 = 360$$
$$5x + 10 - 10 = 360 - 10$$
$$5x = 350$$
$$\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)350$$
$$x = 70$$

Angle measures are $70°$, $71°$, $72°$, $73°$, and $74°$.

    b.   Compare the expressions you used for the five angles and their combined expression. Explain how they are equivalent and how they reveal different information about this situation.

By the commutative and associative laws, $x + (x + 1) + (x + 2) + (x + 3) + (x + 4)$ is equal to $(x + x + x + x + x) + (1 + 2 + 3 + 4)$, which is equal to $5x + 10$. The first expression, $x + (x + 1) + (x + 2) + (x + 3) + (x + 4)$, shows the sum of five unknown numbers where the second is 1 degree more than the first, the third is 1 degree more than the second, and so on. The expression $5x + 10$ shows the sum of five times an unknown number with 10.

MP.2 & MP.7

7.   Let $x°$ be the measurement of an angle. The ratio of the measurement of the complement of the angle to the measurement of the supplement of the angle is $1:3$. The measurement of the complement of the angle and the measurement of the supplement of the angle have a sum of $180°$. Use a tape diagram to find the measurement of this angle.

$$(90 - x):(180 - x) = 1:3$$

1 *unit*

3 *units*

180

4 units $= 180$

1 unit $= 45$

3 units $= 135$
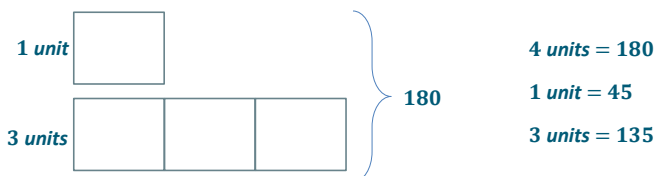
The measurement of the angle that satisfies these criteria is $45°$.

8. **Two lines meet at a point. Set up and solve an equation to find the value of x. Find the measurement of one of the vertical angles.**

A solution can include a modified diagram (as shown) and the supporting algebra work:

$$3x = 117 \qquad\qquad \text{vert. } \angle s$$
$$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)117$$
$$x = 39$$

Each vertical angle: $4(39)° = 156°$

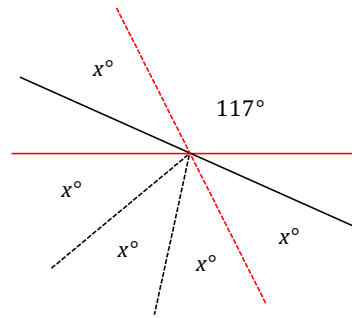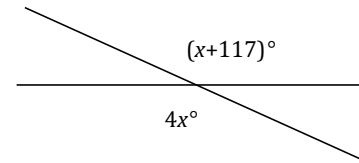*Solutions may also include the full equation and solution:*

$$4x = x + 117$$
$$4x - x = x - x + 117$$
$$3x = 117$$
$$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)117$$
$$x = 39$$

9. **The difference between three times the measurement of the complement of an angle and the measurement of the supplement of that angle is 20°. What is the measurement of the angle?**

$$3(90 - x) - (180 - x) = 20$$
$$270 - 3x - 180 + x = 20$$
$$90 - 2x = 20$$
$$90 - 90 - 2x = 20 - 90$$
$$-2x = -70$$
$$\left(-\frac{1}{2}\right)(-2x) = \left(-\frac{1}{2}\right)(-70)$$
$$x = 35$$

The measurement of the angle is $35°$.

Lesson 4:   Solving for Unknown Angles Using Equations