Lesson 8: Drawing Triangles

Student Outcomes

- Students draw triangles under different criteria to explore which criteria result in many, a few, or one triangle.

Lesson Notes

Students should end this lesson understanding the question that drives Lessons 8–11: What conditions (i.e., how many measurements and what arrangement of measurements) are needed to produce identical triangles? Likewise, what conditions are needed to produce a unique triangle? Understanding how a triangle is put together under given conditions helps answer this question. Students arrive at this question after drawing several triangles based on conditions that yield many triangles, one triangle, and a handful of triangles. After each drawing, students consider whether the conditions yielded identical triangles. Students continue to learn how to use their tools to draw figures under provided conditions.

Classwork

Exercises 1–2 (10 minutes)

Scaffolding:

Students may benefit from explicit modeling of the use of the protractor and ruler to make this construction. Seeing an example of the product and the process aids struggling students.

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Exercises 1–2

1. **Use your protractor and ruler to draw right triangle DEF. Label all sides and angle measurements.**
   a. **Predict how many of the right triangles drawn in class are identical to the triangle you have drawn.**

      Answers will vary; students may say that they should all be the same since the direction is to draw a right triangle.

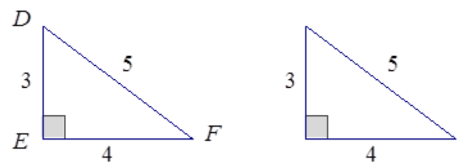
   b. **How many of the right triangles drawn in class are identical to the triangle you drew? Were you correct in your prediction?**

      Drawings will vary; most likely few or none of the triangles in the class are identical. Ask students to reflect on why their prediction was incorrect if it was in fact incorrect.

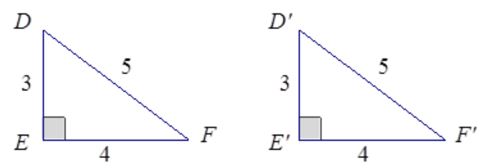
- Why is it possible to have so many different triangles? How could we change the question so that more people could draw the same triangle? Elicit suggestions for more criteria regarding the right triangle.
  - *There are many ways to create a right triangle; there is only one piece of information to use when building a triangle. For people to have the same triangle, we would have to know more about the triangle than just its $90°$ angle.*

Take time at the close of this exercise to introduce students to prime notation.

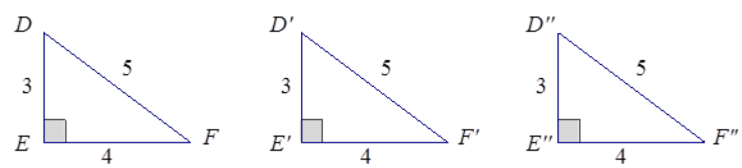
- We use prime notation to distinguish two or more figures that are related in some way. Take, for example, two different right triangles that have equal side lengths and equal angle measures under some correspondence. If the first triangle is $\triangle DEF$ as shown, what letters should we use for the vertices of the second triangle?



- We don't want to use D, E, or F because they have already been used, and it would be confusing to have two different points with the same name. Instead, we could use D', E', and F' (read: D prime, E prime, and F prime). This way the letters show the connections between the two triangles.



- If there were a third triangle, we could use D'', E'', and F'' (read: D double prime, E double prime, and F double prime).



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2. Given the following three sides of $\triangle ABC$, use your compass to copy the triangle. The longest side has been copied for you already. Label the new triangle $A'B'C'$, and indicate all side and angle measurements. For a reminder of how to begin, refer to Lesson 6 Exploratory Challenge Problem 10.

A ____________________ B

B ____________________________ C

A ____ C

A ____ C

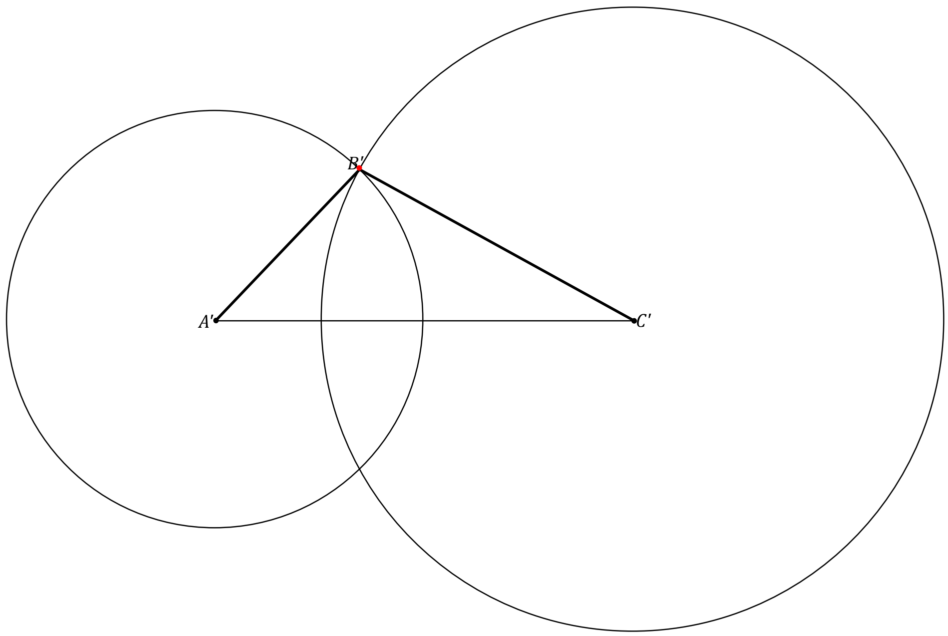
Students must learn how to determine the third vertex of a triangle, given three side lengths. This skill is anchored in the understanding that a circle drawn with a radius of a given segment shows every possible location of one endpoint of that segment (with the center being the other endpoint).

Depending on how challenging students find the task, the following instructions can be provided as a scaffold to the problem. Note that student drawings use prime notation, whereas the original segments do not.

i. Draw a circle with center A' and radius AB.

ii. Draw a circle with center C' and radius BC.

iii. Label the point of intersection of the two circles above $A'C'$ as B' (the intersection below $A'C'$ works as well).



- How many of the triangles drawn in class are identical?
  - *All the drawings should be identical. With three provided side lengths, there is only one way to draw the triangle.*

Exploratory Challenge (25 minutes)

In the Exploratory Challenge, students draw a triangle given two angle measurements and the length of a side. Then, they rearrange the measurements in as many ways as possible and determine whether the triangles they drew are all identical. The goal is to conclude the lesson with the question: Which pieces and what arrangement of those pieces guarantees that the triangles drawn are identical? This question sets the stage for the next several lessons.

The Exploratory Challenge is written assuming students are using a protractor, ruler, and compass. Triangles in the Exploratory Challenge have been drawn on grid paper to facilitate the measurement process. When comparing different triangle drawings, the use of the grid provides a means to quickly assess the length of a given side. An ideal tool to have at this stage is an angle-maker, which is really a protractor, adjustable triangle, and ruler all in one. Using this tool here is fitting because it facilitates the drawing process in questions like part (b).

Exploratory Challenge

A triangle is to be drawn provided the following conditions: the measurements of two angles are $30°$ and $60°$, and the length of a side is 10 cm. Note that where each of these measurements is positioned is not fixed.

a. **How is the premise of this problem different from Exercise 2?**

In that exercise, we drew a triangle with three provided lengths, while in this problem we are provided two angle measurements and one side length; therefore, the process of drawing this triangle will not require a compass at all.

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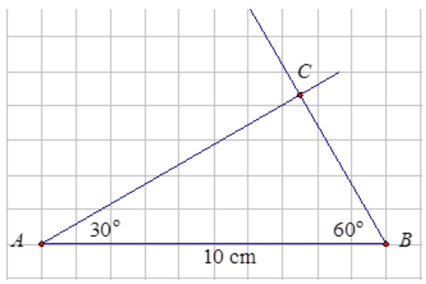
b. **Given these measurements, do you think it will be possible to draw more than one triangle so that the triangles drawn will be different from each other? Or do you think attempting to draw more than one triangle with these measurements will keep producing the same triangle, just turned around or flipped about?**

Responses will vary. Possible response: I think more than one triangle can be drawn because we only know the length of one side, and the lengths of the two remaining sides are still unknown. Since two side lengths are unknown, it is possible to have different side lengths and build several different triangles.

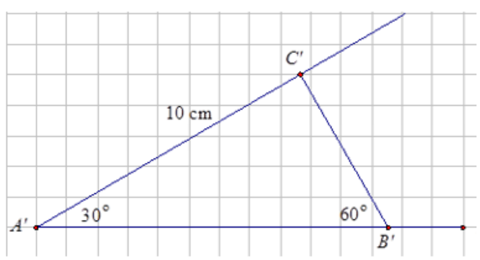
c. **Based on the provided measurements, draw $\triangle ABC$ so that $\angle A = 30°$, $\angle B = 60°$, and $AB = 10$ cm. Describe how the 10 cm side is positioned.**

The 10 cm side is between $\angle A$ and $\angle B$.



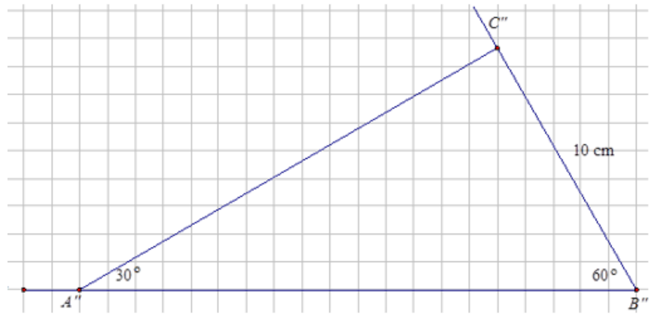
d. **Now, using the same measurements, draw $\triangle A'B'C'$ so that $\angle A' = 30°$, $\angle B' = 60°$, and $AC = 10$ cm. Described how the 10 cm side is positioned.**

The 10 cm side is opposite to $\angle B$.



e. **Lastly, again, using the same measurements, draw $\triangle A''B''C''$ so that $\angle A'' = 30°$, $\angle B'' = 60°$, and $B''C'' = 10$ cm. Describe how the 10 cm side is positioned.**

The 10 cm side is opposite to $\angle A$.



f. **Are the three drawn triangles identical? Justify your response using measurements.**

No. If the triangles were identical, then the $30°$ and $60°$ angles would match, and the other angles, $\angle C$, $\angle C'$, and $\angle C''$ would have to match, too. The side opposite $\angle C$ is 10 cm. The side opposite $\angle C'$ is between 11 and 12 cm. The side opposite $\angle C''$ is 20 cm. There is no correspondence to match up all the angles and all the sides; therefore, the triangles are not identical.

g. **Draw $\triangle A'''B'''C'''$ so that $\angle B''' = 30°$, $\angle C''' = 60°$, and $B'''C''' = 10$ cm. Is it identical to any of the three triangles already drawn?**

It is identical to the triangle in part (d).

h. **Draw another triangle that meets the criteria of this challenge. Is it possible to draw any other triangles that would be different from the three drawn above?**

No, it will be identical to one of the triangles above. Even though the same letters may not line up, the triangle can be rotated or flipped so that there will be some correspondence that matches up equal sides and equal angles.

Discussion (5 minutes)

- In parts (c)–(e) of the Exploratory Challenge, you were given three measurements, two angle measurements and a side length to use to draw a triangle. How many nonidentical triangles were produced under these given conditions?
  - *Three nonidentical triangles*
- If we wanted to draw more triangles, is it possible that we would draw more nonidentical triangles?
  - *We tried to produce another triangle in part (g), but we created a copy of the triangle in part (d). Any attempt at a new triangle will result in a copy of one of the triangles already drawn.*
- If the given conditions had produced just one triangle—in other words, had we attempted parts (c)–(e) and produced the same triangle, including one that was simply a rotated or flipped version of the others—then we would have produced a unique triangle.
- Provided two angle measurements and a side length, without any direction with respect to the arrangement of those measurements, we produced triangles that were nonidentical after testing different arrangements of the provided parts.

- Think back to Exercises 1–2. With a single criterion, a right angle, we were able to draw many triangles. With the criteria of two angle measurements and a side length—and no instruction regarding the arrangement—we drew three different triangles.
- What conditions do you think produce a unique triangle? In other words, what given conditions yield the same triangle or identical triangles no matter how many arrangements are drawn? Are there any conditions you know for certain, without any testing, that produce a unique triangle? Encourage students to write a response to this question and share with a neighbor.
  - *Providing all six measurements of a triangle (three angle measurements and three side lengths) and their arrangement will guarantee a unique triangle.*

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- All six measurements and their arrangement will indeed guarantee a unique triangle. Is it possible to have less information than all six measurements and their respective arrangements and still produce a unique triangle?
  - *Responses will vary.*
- This question guides us in our next five lessons.

Scaffolding:

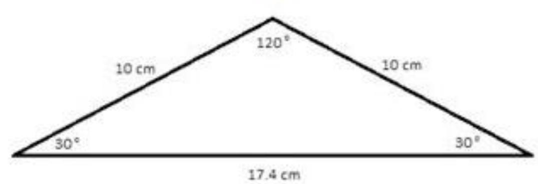
To help students keep track of the conditions that do and do not produce unique triangles, it may be helpful to track the conditions in a chart with examples. Students can add to the chart during the closing of each lesson.

Closing (1 minute)

We have seen a variety of conditions under which triangles were drawn. Our examples showed that just because a condition is given, it does not necessarily imply that the triangle you draw will be identical to another person's drawing given those same conditions. We now want to determine exactly what conditions produce identical triangles.

Have students record a table like the following in their notebooks to keep track of the criteria that determine a unique triangle.

What Criteria Produce Unique Triangles?

Criteria	Example
Three angle measurements and three side lengths	 There is only one triangle with side lengths 10 cm, 10 cm, and 17.4 cm, with angles $30°$, $30°$, and $120°$ as arranged above.

Lesson Summary

The following conditions produce identical triangles:

What Criteria Produce Unique Triangles?

Criteria	Example

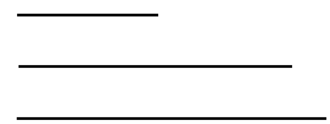
Exit Ticket (4 minutes)

Name ____ Date ____________________

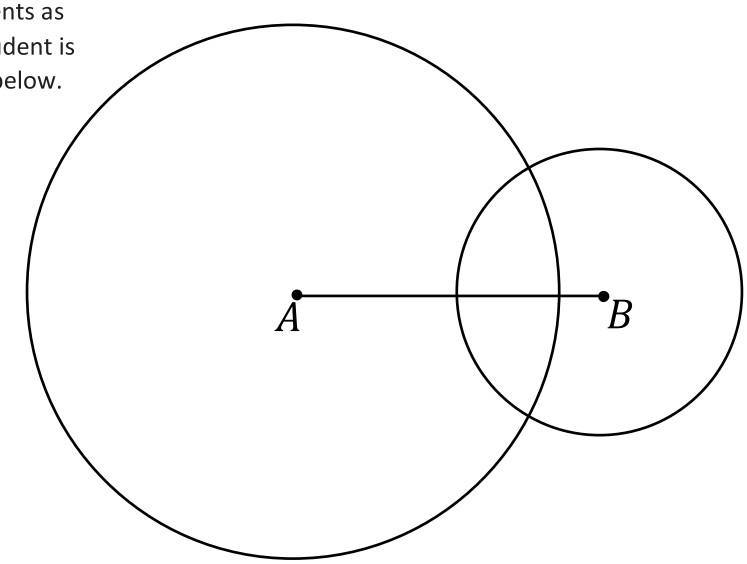
Lesson 8: Drawing Triangles

Exit Ticket

1. A student is given the following three side lengths of a triangle to use to draw a triangle.



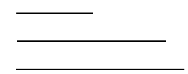
The student uses the longest of the three segments as side $\overline{AB}$ of triangle $\triangle ABC$. Explain what the student is doing with the two shorter lengths in the work below. Then, complete the drawing of the triangle.



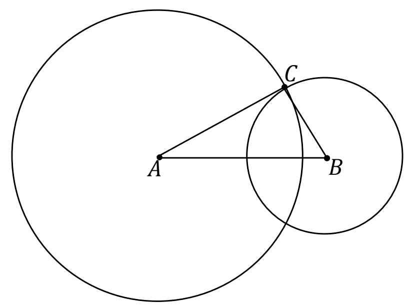
2. Explain why the three triangles constructed in parts (c), (d), and (e) of the Exploratory Challenge were nonidentical.

Exit Ticket Sample Solutions

1. A student is given the following three side lengths of a triangle to use to draw a triangle.



The student uses the longest of the three segments as side $\overline{AB}$ of $\triangle ABC$. Explain what the student is doing with the two shorter lengths in the work below. Then, complete the drawing of the triangle.



The student drew a circle with center A and a radius equal in length to the medium segment and a circle with center B and a radius equal in length to the smallest segment. The points of the circle A are all a distance equal to the medium segment from point A, and the points of the circle B are all a distance equal to the smallest segment from point B. The point where the two circles intersect indicates where both segments would meet when drawn from A and B, respectively.

2. Explain why the three triangles constructed in parts (c), (d), and (e) of the Exploratory Challenge were nonidentical.

They were nonidentical because the two angles and one side length could be arranged in different ways that affected the structure of the triangle. The different arrangements resulted in differences in angle measurements and side lengths in the remaining parts.

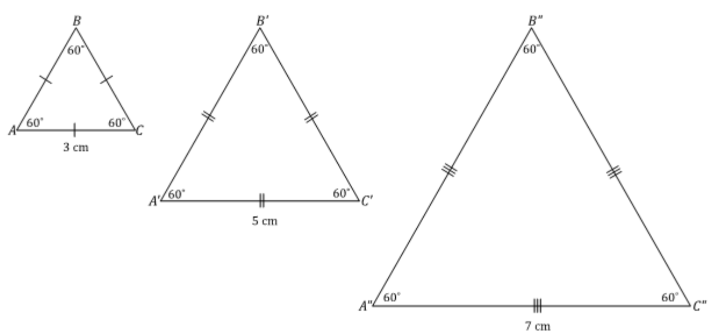
Problem Set Sample Solutions

1. Draw three different acute triangles XYZ, $X'Y'Z'$, and $X''Y''Z''$ so that one angle in each triangle is $45°$. Label all sides and angle measurements. Why are your triangles not identical?

Drawings will vary; the angle measurements are not equal from triangle to triangle, so there is no correspondence that will match equal angles to equal angles.

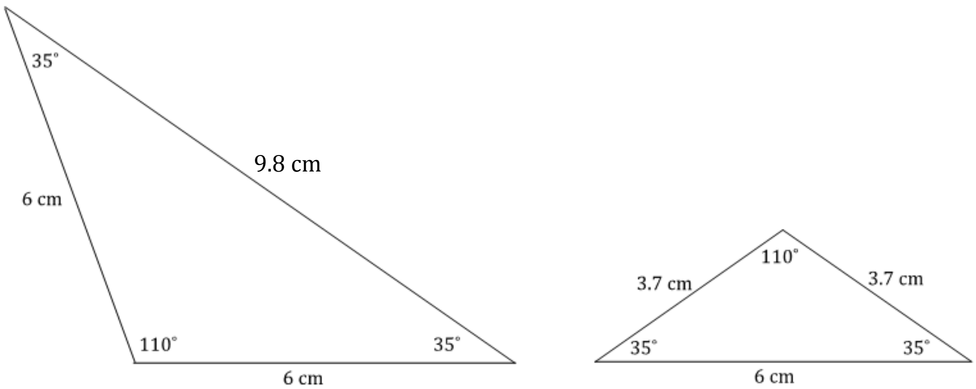
2. Draw three different equilateral triangles ABC, $A'B'C'$, and $A''B''C''$. A side length of $\triangle ABC$ is 3 cm. A side length of $\triangle A'B'C'$ is 5 cm. A side length of $\triangle A''B''C''$ is 7 cm. Label all sides and angle measurements. Why are your triangles not identical?

The location of vertices may vary; all angle measurements are $60°$. Though there is a correspondence that will match equal angles to equal angles, there is no correspondence that will match equal sides to equal sides.



3. **Draw as many isosceles triangles that satisfy the following conditions: one angle measures $110°$, and one side measures 6 cm. Label all angle and side measurements. How many triangles can be drawn under these conditions?**

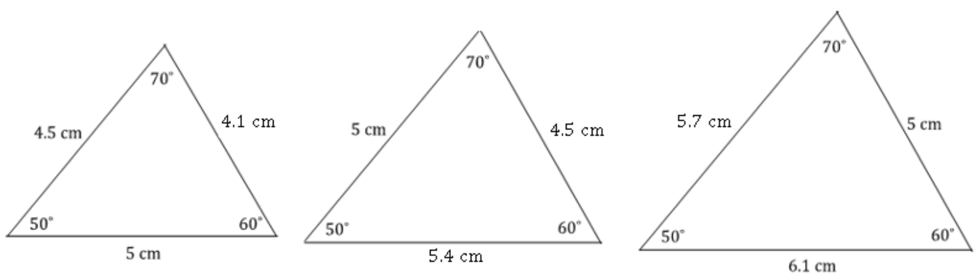
Two triangles



4. **Draw three nonidentical triangles so that two angles measure $50°$ and $60°$ and one side measures 5 cm.**

   a. **Why are the triangles not identical?**

   Though there is a correspondence that will match equal angles to equal angles, there is no correspondence that will match equal sides to equal sides.



   b. **Based on the diagrams you drew for part (a) and for Problem 2, what can you generalize about the criterion of three given angles in a triangle? Does this criterion determine a unique triangle?**

   No, it is possible to draw nonidentical triangles that all have the same three angle measurements but have different corresponding side lengths.